Res. Rep. Fac. Eng. Mie Univ., Vol. 18, pp. 59－72 (1993)

Original Paper

Approximate Method of Elastic Buckling Strength Analysis for Irregular Frames

Shosuke MORINO , Jun KAWAGUCHI
and Hiroko SUZUKI
(Department of Architecture)

(Received September 16 , 1993)

An approximate method is proposed to evaluate the elastic buckling strength of a multi-story multi-bay rectangular frame with irregular arrangements of beam and column stiffnesses and column axial force distribution, combining two methods separately developed by Sakamoto and Wood for single-story multi-bay frames and multi-story single-bay frames, respectively. Sample calculations were made including a 14-story 8-bay frame designed in the real practice, and 6-story 2-bay frames in which one column was extremely slender or subjected to excessive axial force. It is shown that the proposed method gives very good estimates to the buckling strength although the estimates are conservative or unconservative, with no clear tendency.

Key Words : Buckling strength, Multi-story frames, Irregular frames, Approximate method

1. Introduction

The real design practice requires to determine the effective length of a column by the elastic buckling analysis of the overall frame. But its computation is quite cumbersome and time-consuming, and thus an approximate value is used for the effective length, which is derived based on some assumptions. For example, it is usually assumed that the horizontal sway is completely prevented in the case of braced frames, and the effective length is taken equal to the story height as a conservative approximation. For the case of the frame permitted to sway, the effective column length becomes longer than the story height, which is usually evaluated from so-called alignment charts prepared for the effective column length in a rather regular multi-story frame. Exact buckling analysis is rarely performed. Such an alignment chart may cause a great error, if it is applied to the evaluation of the effective column length of a frame in which distributions of column axial forces and member stiffnesses are irregular and unbalanced. This paper presents an approximate method to evaluate the elastic buckling strength of a multi-story frame permitted to sway, which is based on two methods of buckling analysis proposed by others, and investigates the accuracy of this method in view of sample frames.

2. Buckling Strength Computation of Multi-Story Frames

2. 1 Effective Column Length of A Regular Multi-Story Frames

The alignment chart used in the real practice for the evaluation of effective column length in a frame permitted to sway are developed based on the following assumptions : when the frame buckling, i) all columns in the frame buckle simultaneously, and thus the value of the following parameter is identical for all columns ;

$$Z = \sqrt{\frac{Pl_c^{\,2}}{EI_c}} \tag{1}$$

ii) the restraining moments provided by the beams are distributed to the columns above and below the joint in proportion to the column stiffness ; and iii) the rotation angles at both ends of a beam are identical. In Eq. (1), P denotes the axial force, I_c the moment of inertia of a column cross section, E the Young's modulus, and l_c the column length[1).

Based on these assumptions, the slope-deflection analysis of a deformed column due to buckling gives the following equation for the buckling condition[2) ;

$$\frac{G_a G_b \left(\frac{\pi}{\gamma}\right)^2 - 36}{6\,(G_a + G_b)} = \frac{\frac{\pi}{\gamma}}{\tan\frac{\pi}{\gamma}} \tag{2}$$

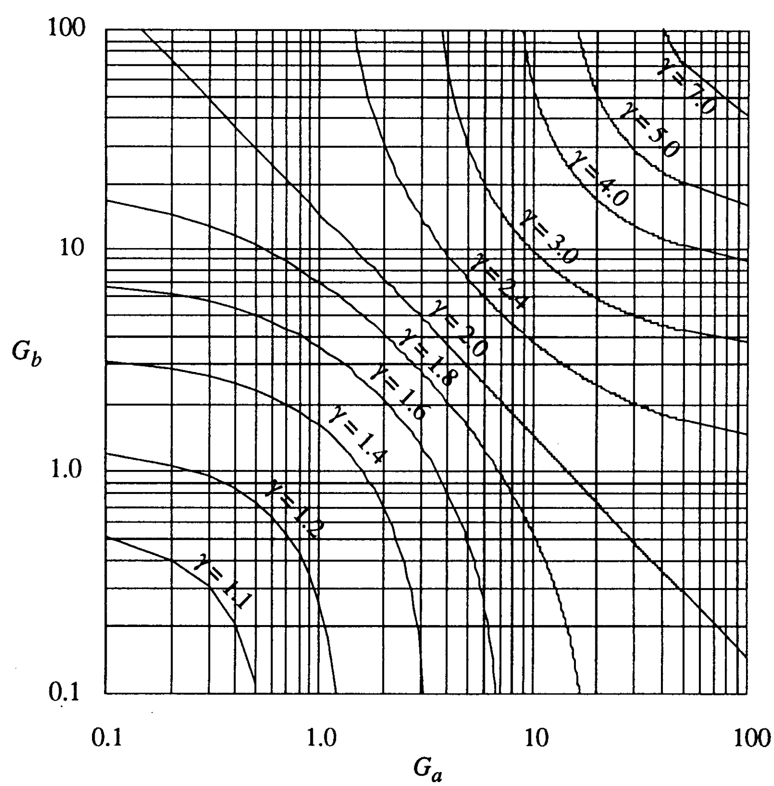


Fig. 1 Alignment chart

$$G = \frac{\Sigma \frac{I_c}{l_c}}{\Sigma \frac{I_g}{l_g}}$$

where γ denotes the effective length factor, and subscripts a and b indicate two end joints of a column ; I and l denote the moment of inertia and length, respectively, and subscripts c and g indicate the column and the beam, respectively. The summation should be taken for all members connected to that joint. The alignment chart shown in Fig. 1 is drawn from Eq. (2). The buckling strength of the column is given by using γ as follows ;

$$P_{cr} = \frac{\pi^2 E I_c}{(\gamma l_c)^2} \tag{3}$$

Equation (2) can be directly derived from the buckling analysis of a simple symmetrical frame shown in Fig. 2, in which the beam stiffness ratios are given as $1 / G_a$ and $1 / G_b$. This means that Eq. (2) expresses the buckling condition of a multi-story frame which is composed of a number of unit frames shown in Fig. 2. Therefore, the alignment chart gives sufficiently accurate results for the case that the values of P_{cr} for all columns in a frame, obtained from Eqs. (2) and (3), are approximately identical. The inaccurate results provided by applying Eqs. (2) and (3) to an irregular frame have been often discussed[3),4)], and modified effective length factor was proposed for rather small scale irregular frames[5)], but an approximate method applicable to general multi-story frames encountered in the real practice has not yet been developed. In the next section, two method proposed by others are shown : the one is applicable to a frame whose distributions of column axial force and member stiffness are regular in the story-direction, but those are irregular in the bay-direction, and the other is applicable to a frame which is regular in the bay-direction, but irregular in the story-direction.

2. 2 Effective Column Length of A Single-Story Multi-Bay Frame - Sakamoto's Method

Sakamoto[3)] presented modified effective column length for a portal frame whose column axial forces and column stiffnesses are different in two columns, as shown in Fig. 3, where α and β denote the axial force ratio and stiffness ratio, respectively, and the effective column length factor $\tilde{\gamma}$ is obtained from the alignment chart shown in Fig. 1. Assuming that the deflected column configurations can be

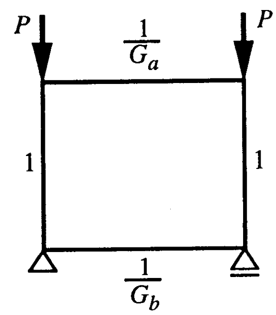


Fig.2. Unit frame for Eq.(2)

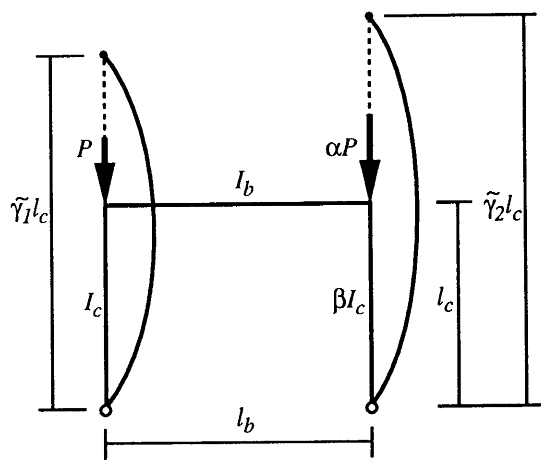


Fig.3. Sakamoto's method

approximated by sine functions with argument $\bar{\gamma} l_c$, the equilibrium of the story shear force for the deformed frame subjected to the axial load P_{cr} gives

$$P_{cr} = \frac{\dfrac{1}{\bar{\gamma}_1^2} + \dfrac{\beta}{\bar{\gamma}_2^2}}{1+\alpha} \cdot \frac{\pi^2 E I_c}{l_c^2} \tag{4}$$

and the modified effective length factor γ_1 and γ_2 are obtained by definition, as follows ;

$$\gamma_1 = \sqrt{\frac{1+\alpha}{1+\beta\left(\dfrac{\bar{\gamma}_1}{\bar{\gamma}_2}\right)^2}}\,\bar{\gamma}_1 \qquad \gamma_2 = \sqrt{\frac{\beta}{\alpha}}\,\gamma_1 \tag{5}$$

This method can be easily extended to a single-story s-bay frame shown in Fig. 4, and the results become as follows ;

$$P_{cr} = \frac{\dfrac{1}{\bar{\gamma}_0^2} + \dfrac{\beta_1}{\bar{\gamma}_1^2} + \cdots + \dfrac{\beta_s}{\bar{\gamma}_s^2}}{1+\alpha_1+\cdots+\alpha_s} \cdot \frac{\pi^2 E I_c}{l_c^2} \tag{6}$$

$$\gamma_0 = \sqrt{\frac{1+\alpha_1+\cdots+\alpha_s}{1+\beta_1\left(\dfrac{\bar{\gamma}_0}{\bar{\gamma}_1}\right)^2+\cdots+\beta_s\left(\dfrac{\bar{\gamma}_0}{\bar{\gamma}_s}\right)^2}}\,\bar{\gamma}_0 \qquad \gamma_i = \sqrt{\frac{\beta_i}{\alpha_i}}\,\gamma_0 \qquad (i = 1, 2, \cdots, s) \tag{7}$$

2. 3 Effective Column Length of A Multi-story Single-Bay Frame - Wood's Method

Wood[6)] presented an exact method of evaluating the buckling strength of a multi-story column elastically restrained by beams at each story joint, as shown in Fig. 5, which is replaced from a multi-story multi-bay frame regular in the bay-direction. This method utilizes the buckling condition that the summation of the modified column stiffness reduced by the axial force and the beam stiffness at any arbitrary joint becomes zero at the instance of buckling, which can be mathematically written as follows ;

$$\Sigma K'' + \Sigma K_b = 0 \tag{8}$$

where K" denotes column stiffness modified by the stability function considering the axial force effect, and K_b the beam stiffness.

Consider the frame shown in Fig. 5, as an example. The value of the axial force P which satisfies Eq. (8) at joint B is searched for by the following procedure. First, the modified stiffness of the top-story column K_3" is evaluated for a trial value of P taking the restraining effect of the beams connected to the joint D. Then, the modified stiffness K_2" of the column BC in a similar manner, taking the restraining effect of the members connected to the joint C, that is, the modified beam stiffness, $\Sigma K_b' = \Sigma K_b + K_3''$. On

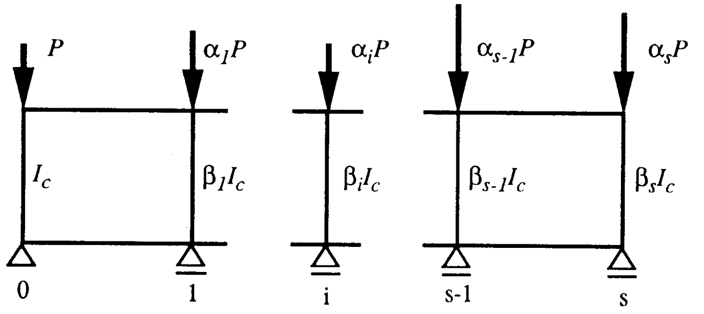


Fig.4. Single-story multi-bay frame

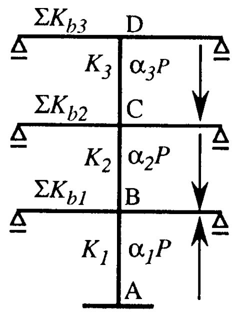


Fig.5. Wood's method

the other hand, the same procedure is taken for the columns below the joint B : evaluation of the modified stiffness K_1'' for the column AB in this case. Then, finally, it is checked whether Eq. (8) is satisfied or not at joint B, by substituting $K_1'' + K_2''$ into $\Sigma K''$, and the beam stiffnesses into ΣK_b. If the value of the left hand side of Eq. (8) becomes negative, the trial value for P already exceeds the buckling strength, and vice versa. In the explanation of the procedure above, the joint B is selected to satisfy Eq. (8), but the choice of the joint is arbitrary ; it could be C.

In the original paper by Wood[6], the equation for the modified column stiffness is given for a column elastically restrained by two beams at a joint, the beam ends being simply supported, as shown in Fig. 5. The modified column stiffness, Eq. (9) given below, is derived for a column in a multi-story single-bay frame shown in Fig. 6 (b) for the convenience in the later computation.

$$K'' = \frac{KZ}{6\tan Z}\left\{1 - \left(\frac{1}{\cos Z}\right)^2 \cdot \left(\frac{\frac{KZ}{6\tan Z}}{\frac{KZ}{6\tan Z} + \Sigma K_b}\right)\right\} \tag{9}$$

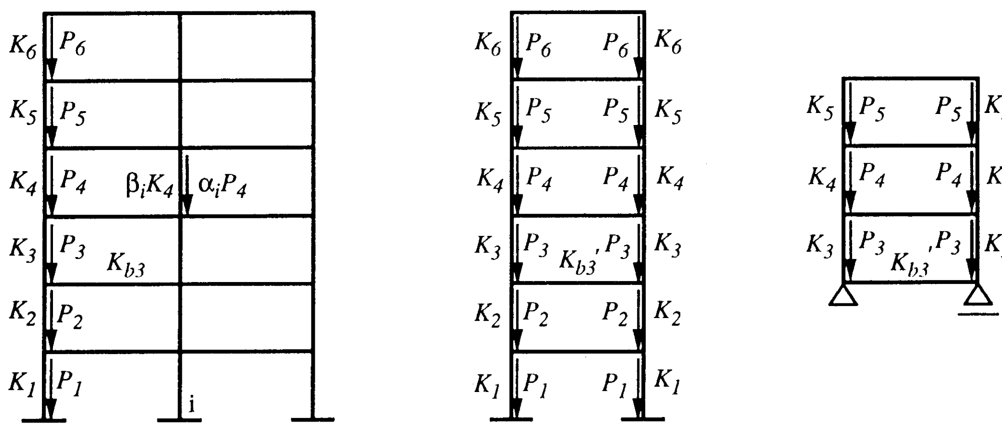
where K denotes the column stiffness ($= I_c / l_c$) , K_b the beam stiffness, and Z is given by Eq. (1).

3. Approximation of Buckling Strength Using A Single-Bay Frame

3. 1 Combination of Sakamoto's and Wood's Method - SWC Method

Sakamoto's and Wood's methods explained in the previous section can be only applicable to a single-story frame or to a single-bay frame, respectively. In this section, a combined method of these two is presented as SWC method to compute the buckling strength of a multi-story multi-bay frame. This method first reforms the prototype frame to a single-bay frame in such a way that the buckling strength of each story of a single-bay frame becomes equal to that of the corresponding story of the prototype frame, both evaluated by Eq. (6). Then, the buckling strength of the single-bay frame is determined by Wood's method, which is finally taken as an approximation of the buckling strength of the prototype frame. The computational procedure is explained below, taking a 6-story 2-bay frame shown in Fig. 6 (a) as an example of the prototype, where member stiffnesses and column axial force ratios are all given.

i) Pick up a column of the prototype frame in Fig. 6 (a) as a reference column, which includes the story



(a) Prototype frame (b) Virtual frame : symmetrical multi-story single-bay frame (c) 3-story partial frame

Fig.6. Method to generate a virtual frame

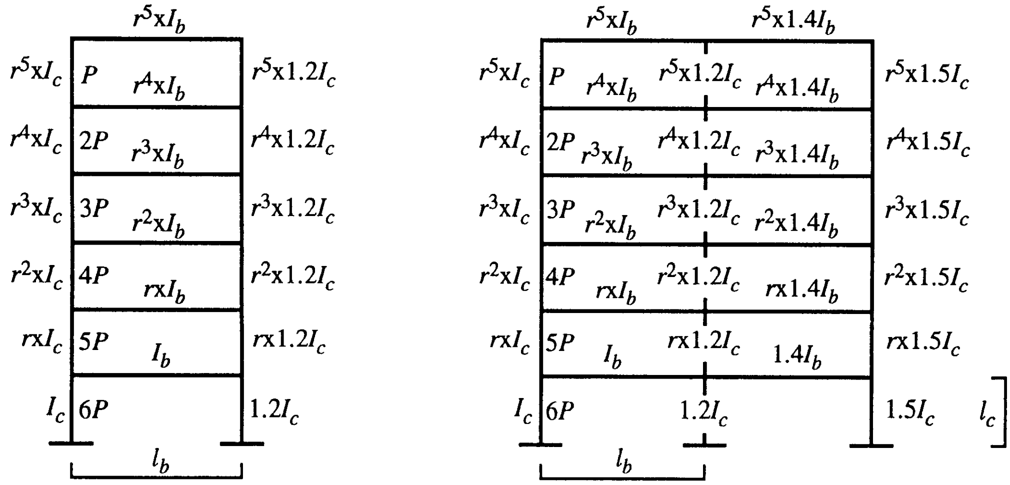
column showing the lowest value of the buckling strength given by Eq. (3) with γ obtained from the alignment chart, Fig. 1 (or by Eq. (2)). The left column is selected as a reference column in this example.
ii) Compute the buckling strength of each story of the prototype frame by Eq. (6) of Sakamoto's method, and compute the effective length factor of the reference (left) column in each story by Eq. (7).
iii) Compose a vertical symmetrical 6-story single-bay frame as shown in Fig. 6 (b), in which the column have exactly the same characteristics as the reference (left) column of the prototype frame ; column axial force ratios and column stiffnesses. The beam stiffnesses of this frame are determined from the 1st story in such a way that the effective length factor becomes equal to that computed in step ii). In this example, since the column base is fixed, G_b is set equal to zero. By entering into the alignment chart (or Eq. (2)) with G_b and the value of γ for the 1st story determined in step ii), G_a can be obtained, from which the modified beam stiffness at the second floor K_{b2}' is determined. The same procedure is repeated to determine the modified beam stiffness at the upper floor levels. The buckling strength of each story of the single-bay frame is thus identical with that of the prototype frame.
iv) Compute by Wood's method the exact buckling strength of the single-bay frame composed above, which is the approximation to the buckling strength of the prototype frame.

The essential part of SWC method is the idea to shrink the prototype frame to the single-bay frame so that Wood's method can be applied to compute the buckling strength, without changing the buckling strength of each story computed by Sakamoto's method.

3. 2 Numerical Examples

(a) Relatively Regular Frames

Taken as numerical examples in this sections are a 6-story single-bay frame (Example 1), a 6-story 2-bay frame (Example 2) and a 14-story 8-bay frame designed in the real practice (Example 3). The axial force ratios and member stiffnesses of these frames are distributed in a relatively regular manner. In the case of the 6-story frames in Fig. 7, the vertical load P is applied at all joints, and thus the column axial



$I_c = 1440 \text{cm}^4$ $I_b = 1152 \text{cm}^4$ $r = 0.8$ $l_b = 600 \text{cm}$ $l_c = 300 \text{cm}$

(a) 6-story 1-bay frame - Example 1 (b) 6-story 2-bay frame - Example 2

Fig.7. Sample frames for buckling strength calculation

forces change as P, 2P, · · , 6P from the top story to the 1st story. The moments of inertia of the columns and the beams reduce from the lower story to the upper story with a proportionality constant of 0.8. The stiffness of the right column of the 6-story single-bay frame in Fig. 7 (a) is 1.2 times that of the left column in each story. In the case of the 6-story 2-bay frame in Fig. 7 (b), the stiffness of the middle and the right columns are 1.2 and 1.5 times that of the left column in each story, respectively, and the stiffness of the right beam is 1.4 times that of the left beam at each floor level. In addition, the following values are taken in the examples : I_c = 1440 cm^4, I_b = 1152 cm^4, l_b = 600 cm, l_c = 300 cm, and E = 205.8 (kN / mm^2). Figure 8 shows the values of member stiffnesses (I_c / l_c , I_b / l_b) of the left column which is selected as the reference column and the adjacent beams, common for two frames in Fig. 7.

Table 1 first shows the buckling strength of each story of sample prototype frames in Fig. 7 computed by Sakamoto's method. Figure 9 shows the member stiffnesses of the virtual symmetrical single-bay frames composed the left (reference) columns of the frames in Fig. 7 (and thus Fig. 8) and the beams whose stiffnesses are so determined that the buckling strength of each story becomes equal to the corresponding value in Table 1. The axial force ratio of the column in each story is the same as the left column of the prototype frames in Fig. 7. Exact buckling strength of the virtual frames in Fig. 9 computed by Wood's method is given in Table 1 as SWC method, together with the ratio in the parenthesis to the value given as exact analysis, which is the result of the analysis of the prototype frames using the slope deflection method with the stability functions, where the effect of the first order sway and bending

Table 1. Buckling strengths by SWC method : Examples 1, 2 and 3 (kN)

Sample frame	Buckling strength of each story by Sakamoto's method						SWC method	Exact analysis
	1st story	2nd	3rd	4th	5th	6th		
Example 1	264.8	116.0	116.0	123.8	148.5	300.1	136.7 (1.013)	134.9
Example 2	331.7	168.3	168.3	179.5	215.4	422.6	189.5 (0.990)	191.5
Example 3	21979	18595	18505	20058	20993	20691	19401 (1.007)	19267

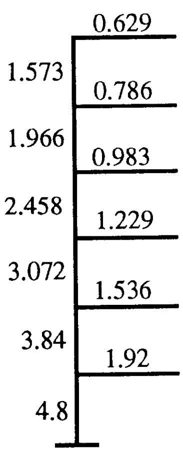
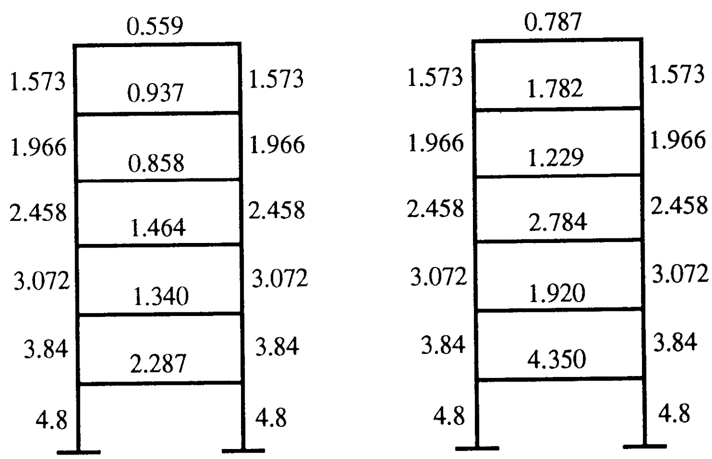


Fig.8. Stiffnesses of left exterior column and adjacent beams



(a) Virtual frame for 6-story 1-bay prototype frame

(b) Virtual frame for 6-story 2-bay prototype frame

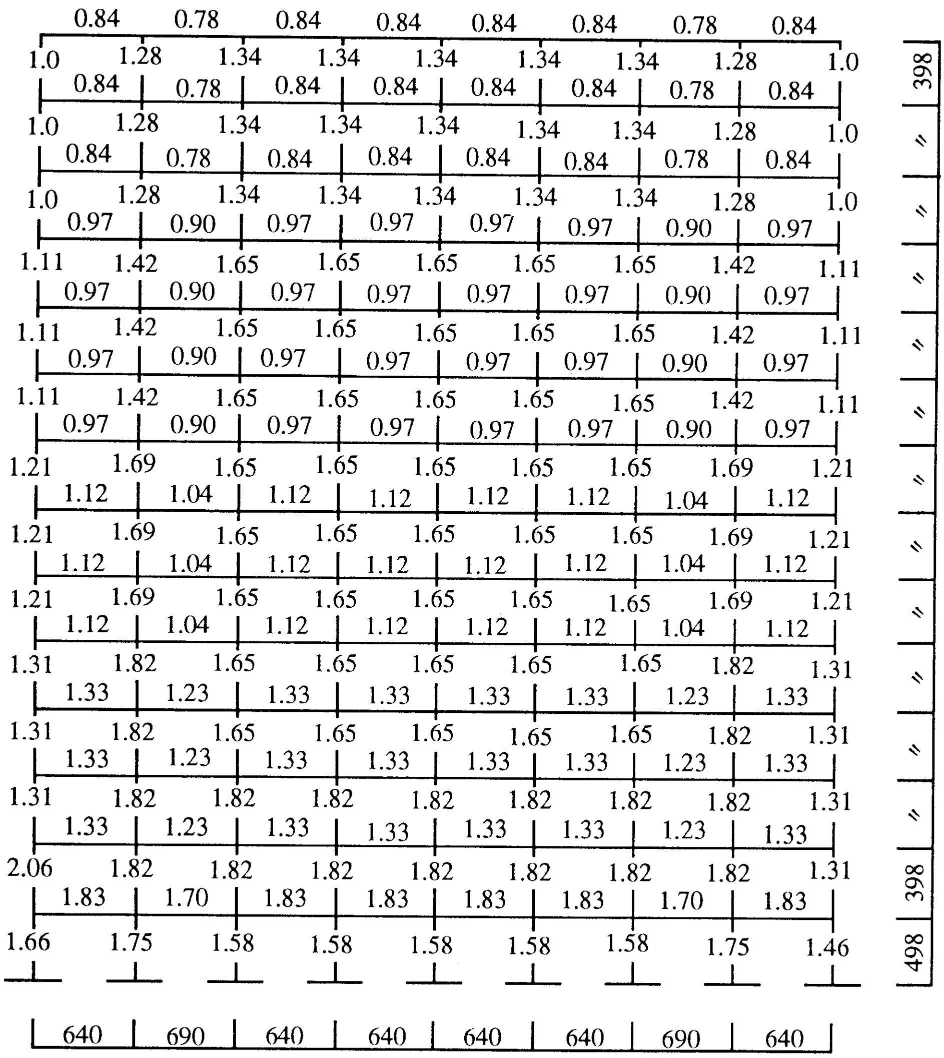
Fig.9. Stiffnesses of virtual frames for SWC method

moments under the application of the vertical loads caused by the irregularity are neglected.

Figure 10 shows the member stiffness ratios and the axial force ratios of the 14-story 8-bay frame designed in real practice, which are the values in reference to the left column in the top story. The results of the numerical computation of the buckling strength of this frame are also shown in Table 1, where the story buckling strengths above the 7th story computed by Sakamoto's method are omitted, since they are all greater than the values listed for 1st to 6th stories.

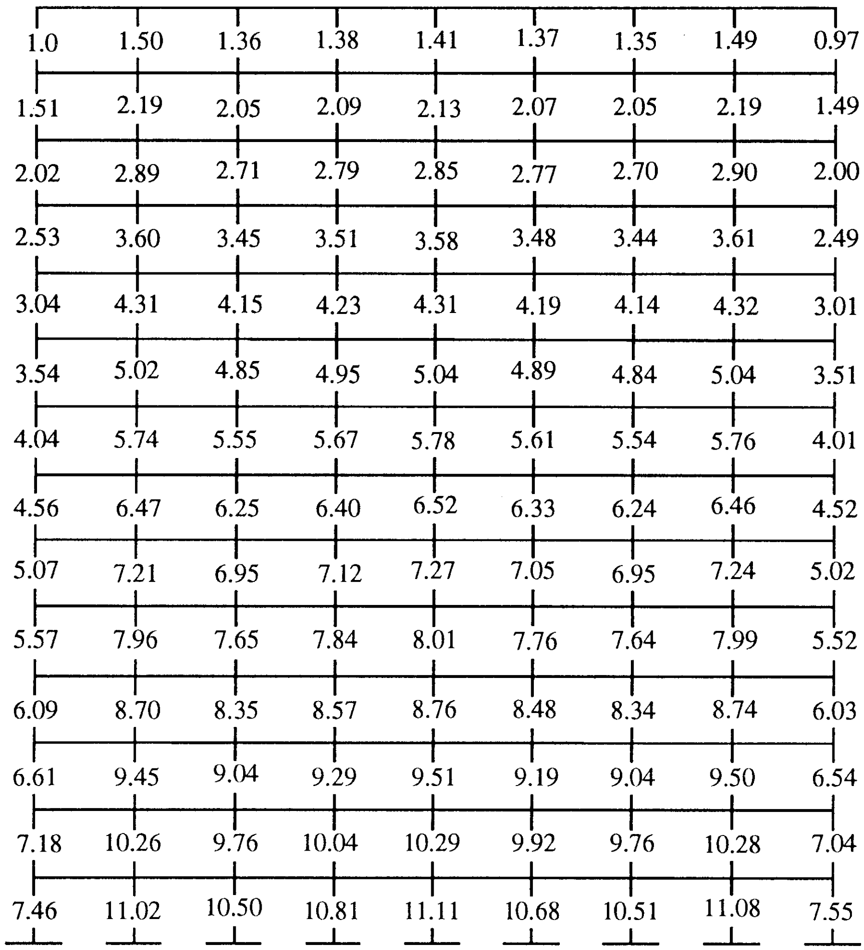
It may be concluded that SWC method possesses high accuracy ; The maximum error observed in three sample frames is only 1.3% as shown in Table 1. The reason of this high accuracy is considered to be as follows :

i) when the axial forces in columns of a frame increase proportionally, the weakest column would reach first its load-carrying capacity, and it is mainly supported by other columns located in the same story, which provide the horizontal sway resistance, and the columns and beams in the upper and the lower stories floor



(a) stiffness ratio

Fig.10. 14-story 8-bay sample frame - Example 3



(b) Axial force ratio

Fig 10. 14-story 8-bay sample frame - Example 3

levels, respectively, which provide the rotational resistance to the columns in the story that the critical column is located. As already well known, the members other than those mentioned above have small effects, or does not provide much restraint to the critical column. In other words, the buckling strength of the overall frame is mainly affected by the characteristics of the members surrounding the critical column. The buckling strength formula for a critical story presented by Sakamoto, Eq. (6), contains all parameters mentioned above.

ii) The virtual symmetrical frame is composed in such a way that the buckling strength of each story of the virtual frame is identical to that of the prototype, both computed by Sakamoto's method. Therefore, if it can be assumed the buckling strength of the overall frame is identical to the buckling strength of a certain critical story locally computed by Sakamoto's method, the buckling strength of the prototype frame is identical to that of the virtual frame. Obviously this assumption is not correct, but we already know that the error may not be so large from the discussion in i).

iii) The characteristics of members not considered in Sakamoto's formula which are located rather far from the critical story can be taken into account, since the virtual frame is solved by Wood's method, which is an exact method.

(b) Frames with Extremely Unbalanced Column Characteristics

The next examples are the same frames shown in Fig.7 (b), but one of the left, middle or right columns in the 4th story is subject to the disturbance, that is, the column stiffness is extremely reduced (Examples 4-1 to 4-3) , or the axial force is extremely increased (Examples 5-1 to 5-3). Figure 11 shows two sample frames : In the one frame, the stiffness of the left column in the 4th story is reduced to the half of that of the frame shown in Fig.7 (b), and in the other frame in Fig.11 (b), the axial forces in the left columns in the 4th and lower stories are increased by 2P, compared with those in the middle and the right columns.

the results of SWC method applied to the frames with unbalanced column characteristics are listed in Table 2, and again the high accuracy is observed ; the maximum error is -3.8% in Example 5-1. The value in the parenthesis indicates the ratio to the exact buckling strength in Table 2 and the following tables.

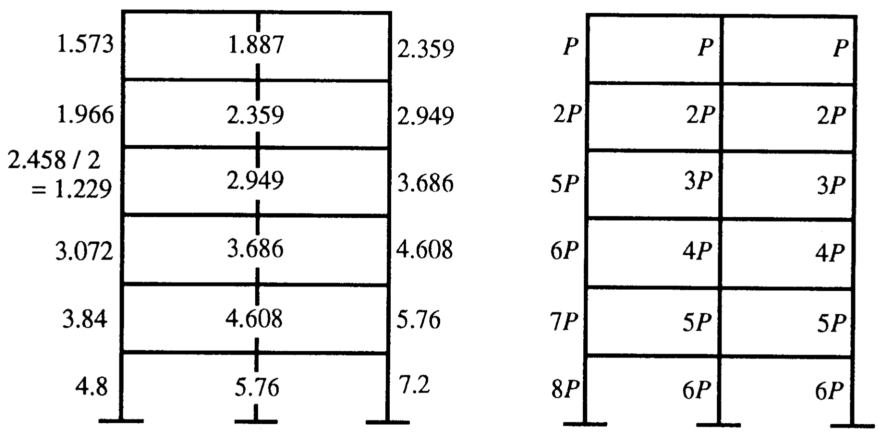
4. Approximate Method of Buckling Analysis

4.1 Approximation by 3-story Partial Frame

The SWC method requires two steps of computation ; composition of a virtual single-bay frame based on Sakamoto's method, and the analysis of the virtual frame by Wood's method. The application of

Table 2. Buckling strengths by SWC method : Examples 4 and 5 (kN)

Sample frame and unbalanced column		SWC method	Exact analysis
4-1	Left col.	183.4 (0.967)	189.6
4-2	Middle col.	182.0 (0.988)	184.3
4-3	Right col.	190.5 (1.008)	188.9
5-1	Left col.	159.0 (0.962)	165.2
5-2	Middle col.	166.3 (1.008)	165.0
5-3	Right col.	167.2 (1.011)	165.3



(a) Sample frame with unbalanced column stiffness

(b) Sample frame with unbalanced column axial force

Fig.11. Sample frames with unbalanced distribution of column stiffness or axial force

this method to large-scale multi-story frames encountered in the real practice is still tedious and time-consuming. From this point of view, two types of approximate method are discussed in this section ; both trying to approximate the prototype frame by a 3-story single-bay partial frame ; the one approximation reduces the partial frame from the virtual symmetrical frame composed in SWC method, and the other reduces the partial frame directly from the prototype frame.

A preliminary computation was performed to investigate the accuracy of the buckling strengths of the 3-story partial frames, using Example 1, 2 and 3. For these 3 examples, the virtual symmetrical frames of single-bay are already composed in the course of computation by SWC method, and then the 3-story partial frames can be simply taken out from the virtual frame as shown in Fig.6 (c), without changing any member characteristics of the virtual frame. A 3-story frame is composed from a reference story and the upper and the lower stories, and therefore, 4 kinds of the 3-story frame can be taken out from a 6-story virtual frame (and thus a 6-story prototype frame), since the 1st and the top story cannot be a reference story.

Table 3 shows the buckling strengths of 4 kinds of the 3-story partial frames for Examples 1, 2 and 3, obtained by Wood's method. For Example 3, the value for the partial frames whose reference stories are 7 to 13 are omitted, since they are all greater than the values listed. If we assume that the smallest buckling strength among those for all the 3-story partial frames give an approximation to the buckling strength of the prototype frame, the results in Table 3 indicates that the approximate buckling strength is given by the strength of the 3-story frame with the 3rd story as a reference in all examples, and the error is still very small, +3.3%. More importantly, note that the smallest value among the story buckling strengths computed by Sakamoto's method also occurs at the 3rd story in all examples, as indicated in Table 1. In other words, the reference story of the 3-story partial frame which gives the smallest buckling strength agrees with the story which gives the smallest story buckling strength computed by Sakamoto's method, in Examples 1, 2 and 3. The values of Approx. method will be explained later.

4.2 Selection Rules for Reference Column and Story

In the course of the approximate method to compute the buckling strength, which will be proposed later, it is required to select a reference column and a reference story to compose a 3-story partial frame which is an approximation of the prototype frame. This section first shows hypothetical selection rules, and then investigates their reliability in view of the numerical results using Examples 4 and 5 shown before.

Selection rules are as follows :

Rule A-Reference column : When composing the virtual single-bay frame, select reference column so that it contains a story column of which buckling strength computed by the alignment chart (or by Eqs. (2) and (3)) is the smallest.

Rule B-Reference story : When composing the 3-story partial frame, select the reference story of which story buckling strength computed by Sakamoto's formula, Eq. (6), is the smallest.

Table 3. Buckling strengths of 3-story partial frames : Examples 1, 2 and 3 (kN)

Sample frame	Reference story						Approx. method	Exact analysis
	1st story	2nd	3rd	4th	5th	6th		
Example 1	-	149.3	139.3 (1.033)	149.1	171.5	-	139.8 (1.037)	134.9
Example 2	-	200.2	193.6 (1.011)	209.5	232.0	-	193.7 (1.012)	191.5
Example 3	-	19703	19617 (1.018)	22001	22257	23551	20384 (1.058)	19267

It has been shown by Example 1 to 3 that the 3-story partial frame composed by applying the selection Rule B gives the smallest buckling strength among all the partial frames taken out from the virtual single-bay frame composed by applying the selection Rule A, and it gives a very good estimates to the exact buckling strength of the prototype frame. In the following, the reliability of the rules is examined with Examples 4 and 5.

Table 4 indicates the results of Examples 4-1, 4-2 and 4-3 already introduced in 3.2 (b) : Frames in which the stiffness of the left, middle or right columns in the 4th story is reduced to half that of the original frame presented as Example 2. In columns (V) and (VI) in Table 4, the reference story column selected according to the Rule A, and the reference story selected according to the Rule B are shown. Then, the buckling strength of the 3-story partial frame composed above according to the rules is given in column (II). The value in column (I), for example 184.4 kN, is obtained as follows : The prototype frame is a 6-story 2-bay frame in which the stiffness of the left column in the 4th story is reduced to the half as shown in Fig.11 (a). First, a virtual single-bay frame is composed using the left column (as indicated in the table) of the prototype frame, and then the buckling strengths of 4 kinds of the 3-story partial frame are computed, among which 184.4 kN corresponding to the the 3-story partial frame with the 3rd story of the virtual 6-story frame as a reference story is the smallest. This numerical procedure is repeated for Example 4-1 changing the reference column as the middle and the right columns of the prototype frame, and 197.1 kN and 200.2 kN are obtained, respectively. Therefore, 12 (4 for each of 3 virtual 6-story frames) different partial frames of 3-stories are examined for Example 4-1, and 184.4 kN marked by a small circle is found to be the smallest, which is the closest to the exact buckling strength given in column (IV). Although not shown in the table, all the values listed in column (I) happened to be for the 3-story partial frames composed

Table 4. Buckling strengths of 3-story partial frames : Example 4, unbalanced stiffness (kN)

Sample frames and unbalanced column		(I) Ref. column			(II)	(III) Approx. method	(IV) Exact analysis	(V) Rule A	(VI) Rule B
		Left	Middle	Right					
4-1	Left col.	○184.4 (0.970)	197.1 (1.039)	200.2 (1.056)	200.1 (1.055)	204.1 (1.077)	189.6	4th story, Left col.	4th story
4-2	Middle col.	187.2 (1.016)	○186.3 (1.011)	194.6 (1.056)	190.0 (1.031)	187.8 (1.019)	184.2	3rd story, Left col.	4th story
4-3	Right col.	191.6 (1.014)	195.8 (1.036)	○189.6 (1.004)	203.1 (1.075)	207.2 (1.097)	188.9	4th story, Right col.	4th story

Table 5. Buckling strengths of 3-story partial frames : Example 5, unbalanced axial force (kN)

Sample frames and unbalanced column		(I) Ref. column			(II)	(III) Approx. method	(IV) Exact analysis	(V) Rule A	(VI) Rule B
		Left	MIddle	Right					
5-1	Left col.	○160.3 (0.970)	172.4 (1.043)	173.9 (1.053)	160.3 (0.970)	153.0 (0.926)	165.2	4th story, Left col.	3rd story
5-2	Middle col.	169.1 (1.025)	○165.4 (1.003)	173.9 (1.054)	169.1 (1.025)	168.7 (1.022)	164.9	3rd story, Left col.	3rd story
5-3	Right col.	○169.1 (1.023)	172.4 (1.043)	169.4 (1.025)	169.4 (1.025)	168.3 (1.018)	165.3	4th story, Right col.	3rd story

with the 3rd story of the virtual 6-story frame as the reference story.

We assumed before that the buckling strength of the 3-story partial frame composed according to the Rules A and B is the smallest among the values for all possible 3-story partial frames, and thus the closest to the exact buckling strength of the prototype frame. The conflicts between the numerical results shown in Table 4 for Examples 4-1to 4-3 and the assumption are as follows :

Example 4-1 : The smallest strength 184.4 kN is for the partial frame with the left column and the 3rd story as the references, and thus the reference story conflicts with the one obtained from the Rule B in column (VI).

Example 4-2 : 186.3 kN is obtained with the middle column and the 3rd story, which conflicts both Rules, A and B given in (V) and (VI), respectively.

Example 4-3 : 189.6 kN is for the frame with the right column and the 3rd story, which conflicts the reference story in (VI).

The same investigation is performed for Examples 5-1 to 5-3, which are the frames having unbalanced distribution of the column axial force as shown in Fig.11 (b), and the results are shown in Table 5. Again, all the values for 9 cases listed in column (I) are obtained for the 3-story frames with the 3rd story of the prototype frame as a reference story. Therefore, no conflict concerning the reference story is found in these examples, but the reference column of the partial frame which gives the smallest buckling strength differs from the one obtained from the Rule A in Examples 5-2 and 5-3.

The investigation above shows that the buckling strength estimated from the 3-story partial frame composed by the Rules A and B is not perfectly reliable. However, the error involved in the values in columns (II) in Tables 4 and 5, which are obtained according to the Rules A and B, is within +7.5% against the exact buckling strength given in column (IV). Thus, we take the Rules A and B as a basis of the final proposed of the approximate method shown in the next section. The values in column (III) will be explained later.

4.3 Proposal of Approximate Method

In the numerical examples shown in the preceding sections, the virtual frame whose story number is the same as that of the prototype frame has to be composed, which sometimes requires a quite amount of computation. Therefore, a procedure to skip the composition of the virtual frame is presented in this section, and proposed as the final method of approximation for the elastic buckling strength of the multi-story frame.

The procedure is quite simple as follows : i) Select the reference column and the reference story according to Rules A and B. ii) Take out the column of 3-story height, as selected in i), without changing the stiffness and axial force distribution from the prototype frame, and compose a 3-story single-bay symmetrical frame, as shown in Fig.6 (c), in which the columns are identical to the one taken out from the prototype frame, and the bottom beam is identical to the corresponding one of the prototype K_{b3}. iii) Determine the stiffnesses of other 3 beams in such a way that the column buckling strength computed by the alignment chart or by Eqs. (2) and (3) becomes identical to the buckling strength computed by Sakamoto's formula, Eq. (6), for the corresponding story. This part of the computation is already explained for SWC method to compose the virtual single-bay frame. iv) Compute the buckling strength of the 3-story partial frame by Wood's method, and it will be the estimate to the buckling strength of the prototype frame.

The results by the approximate method proposed above are listed in column of Approx. method in Table 3, 4 and 5. The maximum error is observed to be +9.7% in Example 4-3, which is a little larger than the values computed by the procedure without skipping the composition of the virtual 6-story frame. However, the error for Example 3, the frame designed in the real practice, is only +5.8% as indicated in Table 3, and it becomes even smaller for relatively regular frames, Examples 1 and 2.

4.4 Limits of Application

Clear limits for applying the proposed method to the real practice cannot be stated yet, but the following points should be noted. The ratio of the maximum value to the minimum value among the buckling strength computed for individual story columns by the alignment chart may be a good indicator for the level of irregularity ; as it becomes large, the gap between the strength by the alignment chart and the exact strength may become large. The ratio observed in the sample frames range from 2.69 (Example 1) to 7.62 (Example 5-1).

The proposed method is based on the assumption that the prototype frame does not contain hinged members, or it does not contain a story or a floor where some columns or beams are missing, respectively. Hinged beams may be easily implemented, but hinged columns may require some special treatment. The discussion presented in the preceding sections omitted the case that the 1st or the top story becomes the reference story. In such a case, the 3-story partial frame may be composed by the bottom 3 stories or by the top 3 stories. These points are left for the future investigation.

Note that both SWC method and the approximate method give conservative or unconservative estimates to the exact buckling strength, and clear tendency has not yet been found. Careful consideration is needed for the application of these method in the real practice.

5. Concluding Remarks

1. The SWC method combining Sakamoto's and Wood's method has high accuracy in estimating the buckling strength of a multi-story frame. The maximum error observed within the numerical examples was -3.8%, and the error for the frame designed in the real practice is only +0.7%.
2. The approximate method proposed to estimate the elastic buckling strength of a multi-story frame, based on a 3-story partial frame composed by the selection rules concerning the reference column and the reference story, contains the maximum error of +9.7% for the frames with unrealistically unbalanced stiffness and axial force distribution, but the error of only +5.8% for the frame designed in the real practice.
3. The approximate method sometimes gives unconservative estimates, thus careful consideration is required when this method is applied to the real practice.
4. The treatment for the frame containing hinged members and open-ceiling type floor arrangement is left for the future investigation.

References

1) Structural Stability Research Council : Guide to Stability Design Criteria for Metal Structures, 3rd Edition, edited by Bruce G. Johnston, John Wiley & Sons, 1976, pp. 418-423.
2) W. McGuire: Steel Structures, Prentice Hall, 1968, pp. 465-474.
3) Architectural Institute of Japan: Guide to Stability Design of Steel Structures, Maruzen, 1980, pp. 224-228.
4) F. Cheong-Siat-Moy: Frame Design without Using Effective Column Length, Journal of the Structural Division, Proceedings of the American Society of Civil Engineers, Vol. 104, No. ST1, 1978. 1, pp. 23-33.
5) K. H. Chu and H. L. Chow: Effective Length in Unsymmetrical Frames, Publications of International Association for Bridge and Structural Engineering, Vol. 29-1, 1969, pp. 1-15.
6) R. H. Wood: Effective Lengths of Columns in Multi-storey Buildings, The Structural Engineer, Vol. 52, 1974. 7, pp. 235-244.