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Original Paper

Analytical Study on Ultimate Strength of Steel–Concrete Composite Sections Under Biaxial Bending

Jun KAWAGUCHI, Shosuke MORINO and Mika UEDA†
(Department of Architecture)

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Abstract

Effects of loading path on the ultimate strength and the moment-curvature relation of composite cross sections subjected to axial load and biaxial bending were investigated by analyzing four kinds of cross section: steel wide-width H-shape, steel square tube, concrete-filled steel square tube and SRC containing steel wide-width H-shape. Three types of loading path were considered: i) monotonically increasing bending moment M_x with constant M_y, ii) proportional loading with a constant ratio of bending moments, M_y/M_x, and iii) proportional deformation with a constant ratio of biaxial curvatures, ϕ_y/ϕ_x. It was found that the same point on the ultimate strength interaction was reached regardless of the loading paths, the maximum values of bending moments may be different from those at the ultimate strength point on the interaction, in the case of the cross section of which flexural strengths about two principal axes were largely different, and the strength deterioration after the maximum strength attained appears most severely in the case of loading type with constant M_y.

Keywords: Biaxial bending, Ultimate strength, Numerical analysis, Moment-curvature relations, Beam-column, Steel-Concrete composite cross section

1. Introduction

Reinforced concrete column with a wide-flange steel encased is frequently encountered in the lower stories of the high rise building, and this type of column is regarded as steel reinforced concrete column. In the previous days, encasing structural steel columns in concrete was a widespread practice for fire protection purpose only, and the increase in stiffness and strength of the column resulting from the encasement was not taken into consideration until some years later. With the development of the theory on the strength of reinforced concrete column section under the axial load and uniaxial bending moment, the ultimate strength of concrete encased steel column under such loading condition has been investigated. It became known that the ultimate strength of a relatively short steel

† General Building Research Corporation

reinforced concrete column under uniaxial compression was equal to the simple sum of the strengths of steel and effective reinforced concrete sections. It was also confirmed by experimental works that the application of the ultimate strength theory on a steel reinforced concrete section under combined axial load and uniaxial bending would estimate well the real strength of such a section.

When a building is subjected to the horizontal loads in an arbitrary direction, biaxial bending moments as well as the axial load act on the column section. In such a case, the ultimate strength of section is expressed as an interaction surface in the general force space, and the ultimate strength theory developed for the uniaxial bending may be extended, although the formulation of the problem is quite complicated.

In 1956, ACI-ASSE Joint Committee 327 published a report summarizing the method of ultimate strength design of reinforced concrete members, where the problem of biaxial bending was not mentioned. Later, Whitney and Cohen (1956) showed the detail of analysis and design of reinforced concrete columns under uniaxial bending based on the ultimate strength, and briefly indicated how to solve the problem of the biaxial bending. AU (1958) seems to be the one that first published the detail of the formulation of the interaction relationship between the axial load and biaxial bending moments. When a rectangular reinforced concrete cross section of subjected to axial force N and bending moments, M_x and M_y, about its principal axes, x and y, respectively, the interaction relation giving the ultimate load carrying capacity may be presented as a surface in the N-M_x-M_y coordinate system. Bresler (1960) tried to approximate the interaction curve between ultimate values of M_x and M_y at constant value of N in the form

$$\left(\frac{M_x}{M_{x0}}\right)^{\alpha} + \left(\frac{M_y}{M_{y0}}\right)^{\alpha} = 1.0 \tag{1}$$

where M_{x0} and M_{y0} are the ultimate moments of the section under the axial load N and uniaxial bending about x and y-axis, respectively, and α is an exponent depending on column dimensions, amount and distribution of steel reinforcement, stress-strain characteristics of steel and concrete, amount of concrete cover, and arrangement and size of lateral ties or spiral. Numerically computed values of α were compared with experimental results. Bresler's work was quoted in Commentary on Building Code Requirements for Reinforced Concrete - ACI 318-63 (1963). Similar works were presented by Furlong (1961) and Pannel (1963), and the latter showed the results of the parametric study on the factors affecting the value of α. One of few experimental results of the ultimate strength of reinforced concrete section under biaxial bending was provided by Meek (1963), and Weber (1966) prepared design charts for the case of diagonal bending of square reinforced concrete sections with symmetrically arranged reinforcing bars in four faces, with amount and arrangement of reinforcing bars being chosen as parameters.

Research works mentioned above all dealt with the reinforced concrete sections under biaxial bending, and assumed Whitney type stress distribution in concrete, i.e., rectangular stress block with $0.85f_c'$, where f_c' is the cylinder strength of concrete. Unfortunately, very little work has been done on the biaxial bending problem of steel reinforced concrete section. Brettle (1971, 1973) presented numerical results for the interaction surface of the case where wide-flange steel was encased in rectangular or circular reinforced concrete section, assuming the stress-strain relation of concrete to be elastic-perfectly plastic. Design charts were furnished in his works in the form of relations between the ultimate axial load and eccentricity with the ration of the steel area to the concrete area as a parameter. Biaxial interaction curves have been numerically investigated by Matsui, Morino and Ueda (1984), Kawaguchi and Morino (1992), and Tsutsui and Sera (1992) for SRC sections containing H- and cross H-steel and concrete-filled steel tubular (CFT) section, and the value of the parameter a in Eq. (1) was discussed. As to the experimental investigation of the biaxial interaction curves, very few work has been done except for the work by Morino, Matsui and Watanabe (1984) and Morino, Uchida and Ozaki (1987). Thorough reviews of the general research works concerning the steel reinforced concrete members were given by Mcdevitt and Viest (1972, a and b) and Wakabayashi, Naka and Kato (1972).

As revealed in the course of the literature survey, no research was found which mentioned the effect of loading procedure, although some tests and analysis about the column subjected to biaxial bending have been carried out, and the beneficial data have been obtained, as described above. The purpose of this investigation is to clarify the effect of the loading procedure on the ultimate strength by numerical analysis which is based on the moment-curvature relation considering the strain reversal and the local buckling of steel elements.

2. Method of Analysis

2.1 Analytical model

The wide flange, the square hollow section, the concrete-filled steel tube and the wide flange section encased in the concrete were investigated. The shapes and the dimensions of the cross sections are shown in **Fig. 1**. The cross sections were assumed to be formed by the straight plate elements, and no curved part existed. The stress-strain relations of steel, reinforcements and concrete were assumed as shown in **Fig. 2**. The relation of the steel contains the stress reduction part caused by the local buckling on the compressive side, which was expressed by the authors (1991) in detail, and those of the reinforcement and the concrete were expressed by the bilinear shape. The strain hardening of the reinforcement were considered. The tensile stress was neglected and the ultimate

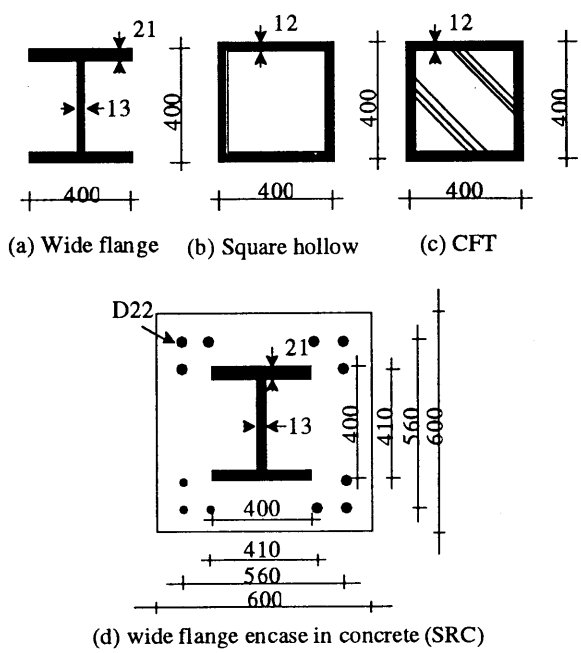

Fig. 1 Cross-sections

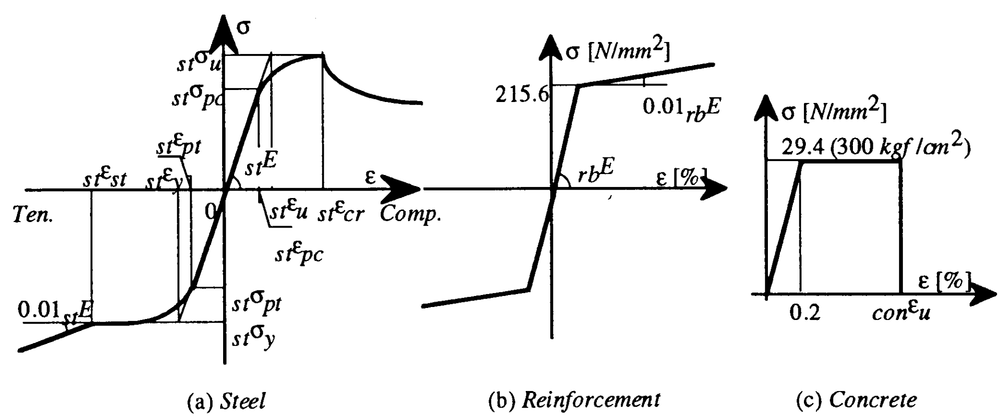

Fig. 2 Stress-strain relations of material

strain were set equal to 0.4 or 0.6 % in the case of the concrete. Materials assumed here were SM490 class (F=323.4 N/mm^2) for steel, SD40 class (F=211.6 N/mm^2) for reinforcement, and the 300 kgf/cm^2 for cylinder strength concrete. Other parameters for the stress-strain relations are shown in **Table 1**.

Table 1 Parameters

$_{st}E(kN/mm^2)$	205.8
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$_{st}\sigma_{pc}(N/mm^2)$	196
$_{st}\sigma_{pt}(N/mm^2)$	-196
$_{st}\sigma_{y}(N/mm^2)$	-323.4
$_{st}\sigma_{u}(N/mm^2)$	288.6
$_{st}\varepsilon_u$ (%)	0.14
$_{st}\varepsilon_{cr}$ (%)	0.67
$_{con}\varepsilon_u$ (%)	0.6(CFST) 0.4(SRC)

2.2 Moment-curvature relations

The moment-curvature relations about each major axis were calculated for the cross-section subjected to the axial load and the biaxial bending. On the assumption that the plain of the cross-section remains plain, the strain distribution in the cross-section subjected to the axial load N and the biaxial bending moments M_x and M_y are as shown in **Fig. 3.** Suppose that the strain at the center of gravity of the cross-section is denoted by ε_0, the curvature about the bending axis by ϕ, and the inclination angle of the bending axis against the x-axis by θ_f, then the curvatures ϕ_x and ϕ_y about x- and y-axis are given as

$$(\phi_x, \phi_y) = (\phi \cos\theta_f, \phi \sin\theta_f) \tag{2}$$

Thus, the strain ε_i at a reference point in the cross-section (x_i, y_i) is given as

$$\varepsilon_i = \varepsilon_0 + \phi_y x_i \quad \phi_x y_i \tag{3}$$

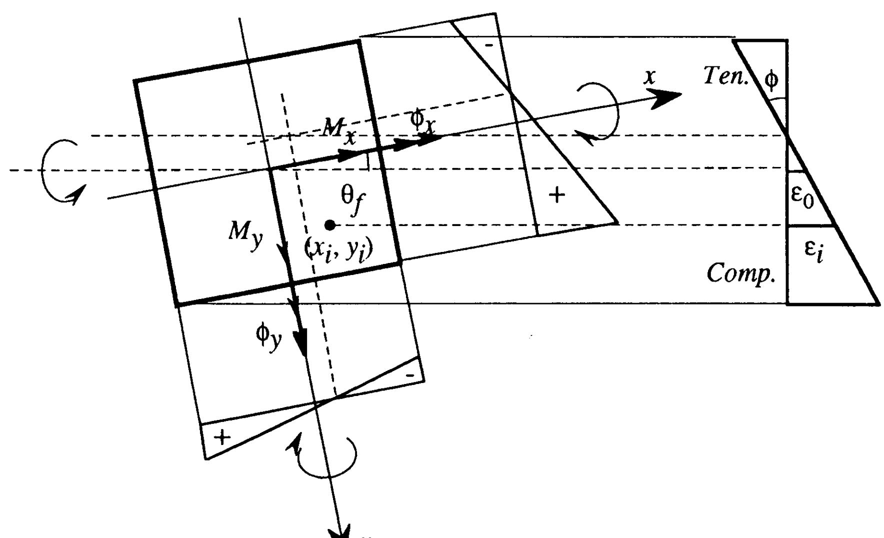

Fig. 3 Strain distribution

where the vectors of moments and curvatures are positive when they are directed to the positive directions of x- and y-axes, and the compressive strain is taken positive. The stress distribution can be directly obtained from the strain distribution. First, the cross-section is divided into small elements and the stress is assumed to be uniformly distributed in each element. Then, the axial load N and the bending moment about x- and y-axis (M_x, M_y) can be calculated by numerical integration as follows:

$$\begin{aligned} N &= \Sigma\, a_i\, \sigma_i \\ M_x &= \Sigma\, a_i\, \sigma_i\, y_i \\ M_y &= \Sigma\, a_i\, \sigma_i\, x_i \end{aligned} \tag{4}$$

where a_i : the cross-sectional area of element;

The moment-curvature relations about each axis of the four kinds of cross-section are calculated according to the following three loading procedures.

A: keep the moment about y-axis to constant, and gradually increase the moment about x-axis.
B: keep the deformation direction ($\theta_f = \phi_y / \phi_x$) to the value which is obtained at the ultimate point in the procedure **A**, and gradually increase moment.

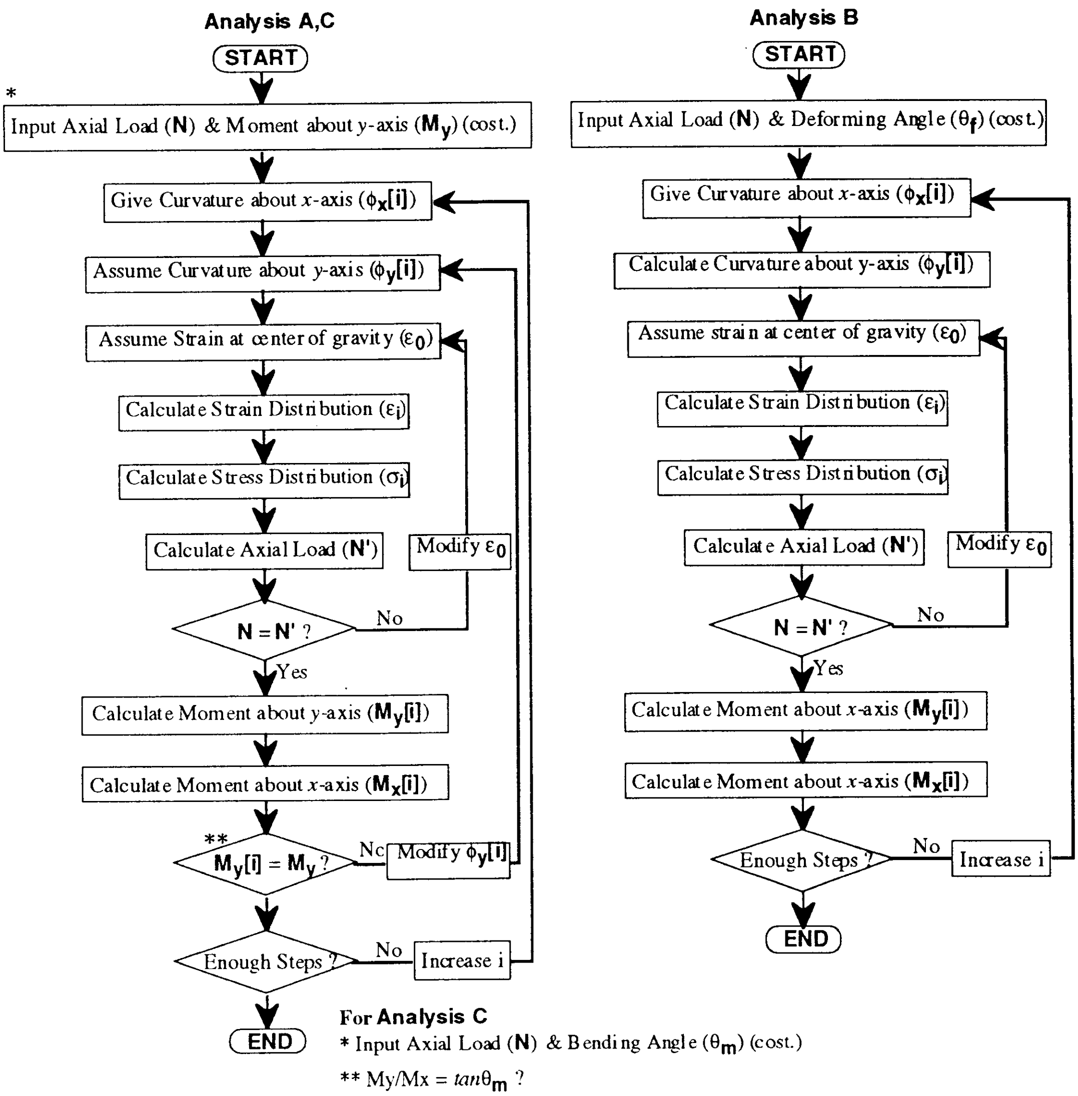

Fig. 4 Flow charts of analysis

C: keep the bending direction ($\omega\mu = M_y / M_x$) to the value which is obtained at the ultimate point in the procedure **A**, and gradually increase moment.

The flow charts of the analysis are shown in **Fig. 4**. In the procedure **A**, the axial load N and the moment about y-axis M_y were kept to the certain value by controlling the strain at the center of the gravity (ε_0) of the cross-section and the curvature about y-axis ϕ_y, while the moment increased. After ε_0 and ϕ_y were determined, the strain and stress distribution were computed. Then, the moment M_x and M_y were calculated. The trial and error procedure between N and ε_0, and between M_y and ϕ_y were solved by using the *Newton-Raphson* method. In analysis **C**, the value ω_m ($= M_y / M_x$) was taken in the trial and error procedure instead of M_y in the analysis **A**.

In the analysis **B**, if the curvature about x-axis ϕ_x is given, the curvature about y-axis ϕ_y is simply determined by the following equation:

$$\frac{\phi_y}{\phi_x} = tan\ \theta_f \tag{5}$$

Then, the only condition to satisfy is the equilibrium of the axial load.

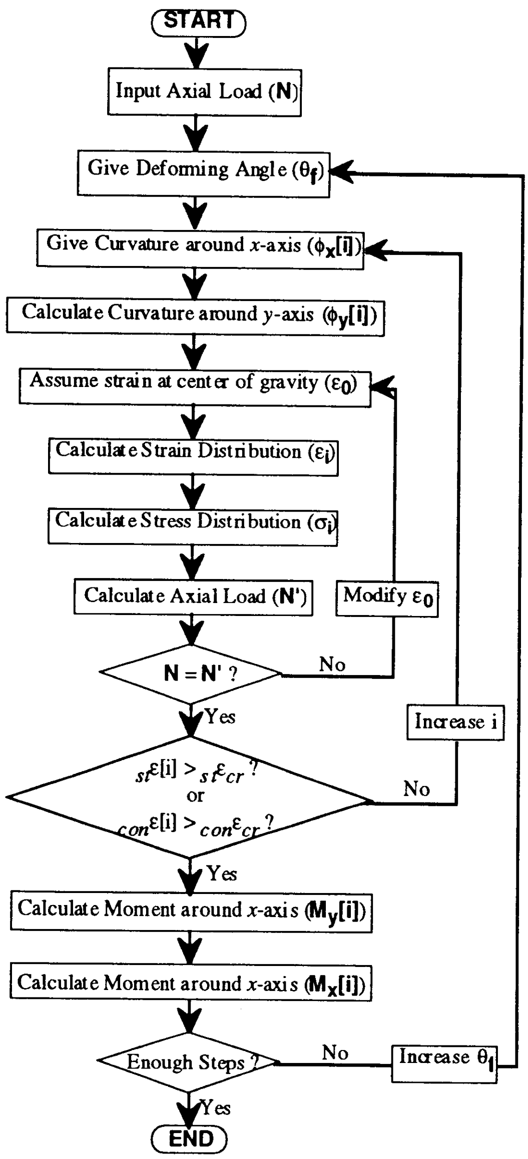

Fig. 5 Flow chart for ultimate strength curve

In all analysis, the calculation was proceeded by increasing the curvature about x-axis. The value of M_y which had been kept constant was set to the value equal to the long-term allowable bending moment calculated by Standards for steel structures (1973) or SRC structures (1987).

2.3 Ultimate Strength

The ultimate strength was calculated under the condition that the curvature about the bending axis increased until the any element of compressive side reached to the strain of the local buckling of the steel or the squash strain of the concrete. Then, the relation between M_x and M_y was obtained by changing the bending axis. The flow chart of this procedure is shown in **Fig. 5**.

3. Analytical Results and Discussion

Figure 6 shows the M_x-M_y relations of cross-section obtained from previous analysis. The analysis **A** in which M_y kept constant, and the analysis **C** in which the bending direction kept to the constant show the linear relation from the beginning to the ultimate strength on the interaction curve. On the other hand, analysis **B** shows the curved relation. However, the ultimate points of all analysis met the exactly same point on the interaction curve for the ultimate strength of the cross-section subjected to the axial load and biaxial bending.

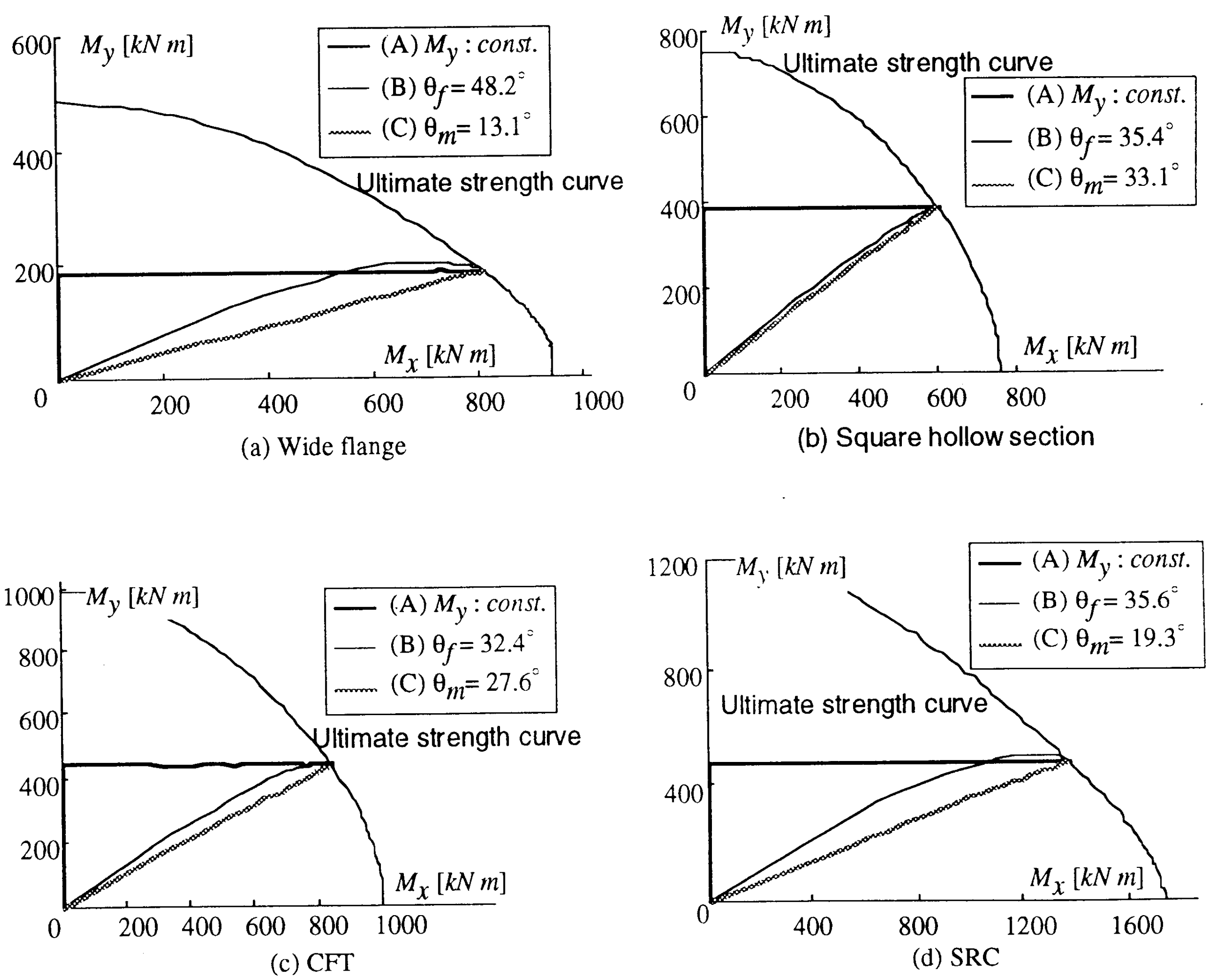

Fig. 6 Biaxial Ultimate Strength

The maximum values of M_x and M_y of the wide flange and the wide flange section encased in concrete are different from the ultimate strengths given by the interaction curves. This is because these cross sections have unequal strength about each major axis.

Figure 7 shows the moment-curvature relations about each major axis obtained from the analyses **A**, **B** and **C**. The black triangles show the points where the strength of the cross-section reached the interaction curve for the ultimate strength. From these figures, the following facts were observed: The peak points of all moment-curvature relations obtained from these analysis met at the identical point; the initial stiffness of M_x-ϕ_x relations of the analysis **C** was the largest; and the strength deterioration after the peek point of the analysis **A** was the largest. As mentioned before, the maximum values of M_x and M_y appeared after and before the ultimate strength point on the interaction curve, respectively in the analysis **B** of the section having unequal strength.

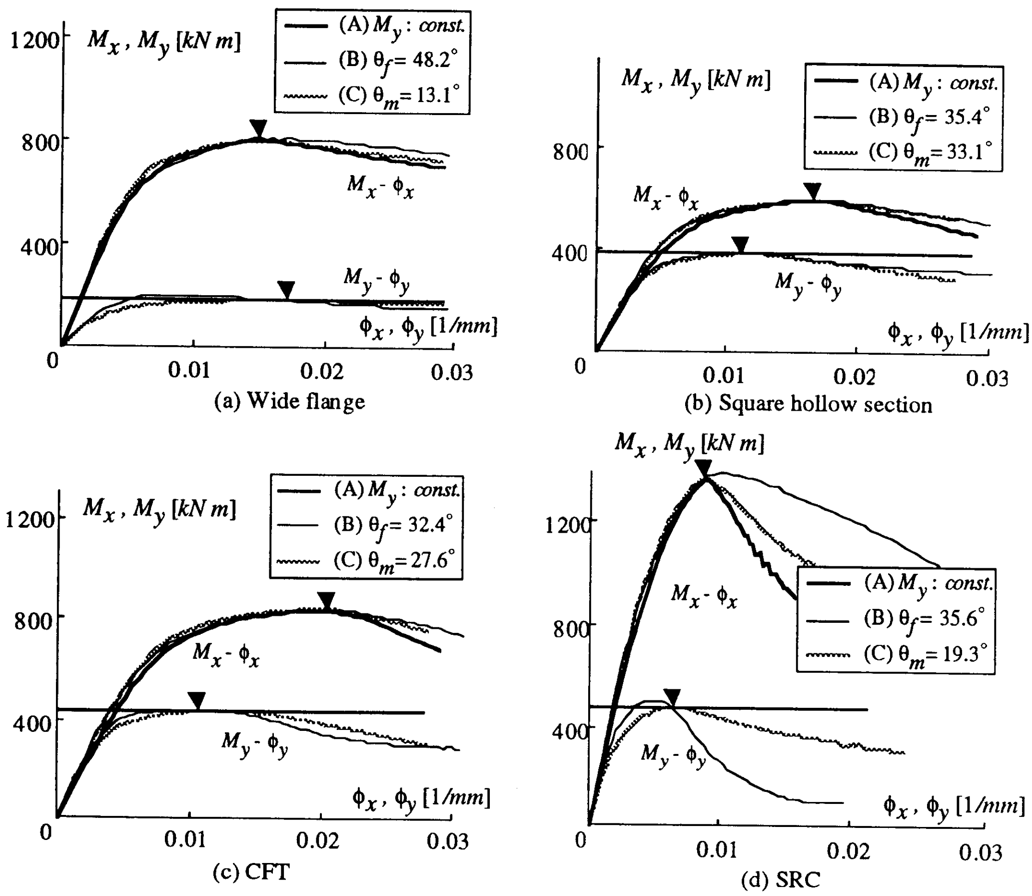

Fig. 7 Moment-curvature relations

4. Conclusion

1) The identical point on the interaction curve for the ultimate strength of the cross-section subjected to the axial load and the biaxial bending moment was reached regardless of the loading procedure.

2) The moment-curvature curves obtained from three different analyses met at one point, and the value of the moment at this point is on the interaction curve for the ultimate strength.

3) The ultimate strength point on the interaction curve was not the same as the maximum point of M_x and M_y in the case of the analysis of deformation angle kept constant (Analysis **A**) for the section having unequal strength about each major axis.

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