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Original Paper

Elastic Buckling Strength of Semirigid Frames

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Abstract

The behavior of semirigid frames has been extensively investigated, and several methods has been proposed for the evaluation of effective buckling length of framed columns, which require rather tedious calculation in the case of the connection with highly nonlinear characteristics. This paper proposes an approximate method to evaluate the effective length of columns involved in a frame of which connections can be assumed to behave linearly. The proposed method first replaces a given frame with elastic linear springs at beam ends by an equivalent rigid frame, and the effective column length of the former is approximated by that of the latter, which may be obtained from a conventional alignment chart. Good accuracy has been observed from the results of the numerical analysis of sample frames.

Keywords: semirigid frame, elastic buckling, elastic springs, appropximate solution

1. Introduction

Recently, the behavior of frames with semirigid connections has been extensively investigated experimentally and theoretically, and the frame analysis considering semirigidity is carried out in some cases of the real design practice. Since the semirigid connection shows nonlinear behavior in the early stage of loading, nonlinear analysis is required to evaluate effective buckling length of framed columns, eventhough the frame is within the elastic range, which is rather tedious. Nethercot, et al. applied B-spline model to the moment-rotation relation of an H-shaped beam-to-column connection using end-plates or angles, and showed the column curves for the columns with semirigid end restraints[1]. Chen, Kishi, Goto, et al. presented a method of buckling analysis of frames with connection rigidities determined by beam-line method[2], and showed the results of tangent modulus analysis of sway-prevented[3] and sway-permitted[4, 5] fish-bone type multi-story frames, of which connection rigidity was given by a power model[6]. These works investigated the applicability of alignment charts to semirigid frames, by comparing exact effective buckling lengths of framed columns with those obtained from the alignment charts. Tsuji, Ohtani, et al. invesigated the effects of strength and rigidity of connections on the collapse types of portal frames, i.e. mechanism-forming collapse or instability collapse, in which truss model and rotational spring model were used to simulate shear deformation and local deformation of a connection panel, respectively[7]. Matsui, Kawano, et al. investigated the effects of degree of fixity of the column base on the buckling strength of frames, by modeling the column base by a rotational spring[8].

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In general, the beam-to-column connection consisting of H-shaped members with end-plates or angles shows strong non-linearity, but a linearity is recognized in some types of connections, such as the connection at which an H-shaped beam is welded to a pipe column without diaphragm, and a bare type column base with relatively-high fixity. Therefore, the connection was sometimes treated as a linear element[9, 10]. If the behavior of a semirigid connection can be assumed bi-linear, conventional allowable stress design or plastic design is applicable to semi-rigid frames, and it is needed to evaluate the column effective length in the course of column design. Along this context, this paper rpalaces a frame with semirigid connections by a frame with linear rotational springs at beam ends, and proposes an approximate method to evaluate column effective length of the frame with beam end springs, utilizing conventional alignment charts[11]. Accuracy of the approximate method is investigated with the exact solution of several sample frames.

2. Elastic Buckling Analysis of Semirigid Rectangular Frames

2.1 Model for the Analysis

Figure 1(a) shows a model for the analysis, which is a rectangular frame with linear springs at each beam end, subjected to vertical load P on each column. Column stiffness and stiffness of upper and lower beams are denoted by K_c, K_b and K_f, respectively, and rotational spring constants of springs at upper and lower beam ends are denoted by K_{bs} and K_{fs}, respectively. Member stiffness K is defined by $K = EI/l$, where E, I and l denote Young's modulus, moment of inertia and length of the member, respectively. Figure 1(b) shows the same frame in which k_b, k_f, k_{bs} and k_{fs} denote the stiffness ratios to the column stiffness of the upper and lower beams, and the springs at the upper and lower beam ends, respectively. The stiffness ratio of a spring is defined here as the rotational spring constant divided by the column stiffness, for example, $k_{bs} = K_{bs}/K_c$.

2.2 Exact Analysis

The slope-deflection formulas for the member-end moments of a beam with springs at both ends in which chord rotation does not occur, M_{bc} and M_{cb} of the beam BC in Fig. 1(b), for example, are expressed as follows[12]:

$$\begin{aligned} M_{bc} &= k_b K_c (k_{bb}\theta_b + k_{bc}\theta_c) \\ M_{cb} &= k_b K_c (k_{cb}\theta_b + k_{cc}\theta_c) \end{aligned} \tag{1}$$

where

$$\begin{aligned} k_{bb} &= \frac{12(k_{bs1}k_{bs2} + 3k_b k_{bs1})}{4(k_{bs1}+3k_b)(k_{bs2}+3k_b) - k_{bs1}k_{bs2}} \\ k_{bc} &= \frac{6k_{bs1}k_{bs2}}{4(k_{bs1}+3k_b)(k_{bs2}+3k_b) - k_{bs1}k_{bs2}} \\ k_{cc} &= \frac{12(k_{bs1}k_{bs2} + 3k_b k_{bs2})}{4(k_{bs1}+3k_b)(k_{bs2}+3k_b) - k_{bs1}k_{bs2}} \end{aligned} \tag{2}$$

On the other hand, the slope-deflection formulas for the member-end moments and lateral forces of a column subjected to the axial force in which chord rotation occurs, M_{ab}, M_{ba}, V_{ab} and V_{ba} of the column AB in Fig. 1(b), for example, are expressed as follows[12]:

$$\begin{aligned} M_{ab} &= K_c(\alpha\theta_a + \beta\theta_b - \gamma R) \\ M_{ba} &= K_c(\beta\theta_a + \alpha\theta_b - \gamma R) \end{aligned} \tag{3}$$

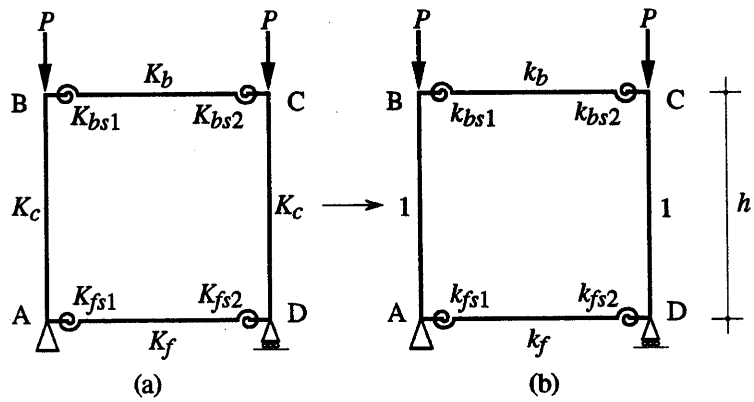


Fig. 1 Model for Analysis: Semirigid Rectangular Frame

$$V_{ab} = -\frac{K_c}{h}\,(\gamma\theta_a + \gamma\theta_b - \delta R)$$

$$V_{cd} = -\frac{K_c}{h}\,(\gamma\theta_c + \gamma\theta_d - \delta R) \tag{4}$$

In the above equations, α, β, γ, and δ are stability functions of the load parameter Z, and given by

$$\alpha = \frac{Z\,sinZ - Z^2 cosZ}{2(1-cosZ) - Z\,sinZ}$$

$$\beta = \frac{Z^2 - Z\,sinZ}{2(1-cosZ) - Z\,sinZ}$$

$$\gamma = \alpha + \beta = \frac{Z^2(1 - cos\ )Z}{2(1-cosZ) - Z\,sinZ} \tag{5}$$

$$\delta = 2\gamma + Z^2 = \frac{Z^3 sinZ}{2(1-cosZ) - Z\,sinZ}$$

where

$$Z = h\sqrt{\frac{P}{EI}} \tag{6}$$

Figure 2(a) shows the deformations occurring when the frame shown in Fig. 1(b) buckles with sidesway. In this case, equilibrium of bending moments acting around 4 panel points A, B, C, and D, and equilibrium of story shear lead to 5 equations, and eliminating the chord rotation of the column R leads to a set of homogeneous simultaneous equations of size 4, with the unknowns θ_a, θ_b, θ_c and θ_d. In the case of the buckling without sidesway shown in Fig. 2(b), the equilibrium of story shear is not necessary, since the chord rotation does not appear, and a similar set of homogeneous simultaneous equations is obtained.

Details of derivation of the simultaneous equations which gives the buckling condition of a frame are shown in Ref. [12]. Only the final buckling conditions are shown below.

Bckling with sidesway:

$$\begin{bmatrix} k_f k_{aa} + \alpha - \frac{\gamma^2}{2\delta} & \beta - \frac{\gamma^2}{2\delta} & -\frac{\gamma^2}{2\delta} & k_f k_{ad} - \frac{\gamma^2}{2\delta} \\ \beta - \frac{\gamma^2}{2\delta} & k_b k_{bb} + \alpha - \frac{\gamma^2}{2\delta} & k_b k_{bc} - \frac{\gamma^2}{2\delta} & -\frac{\gamma^2}{2\delta} \\ -\frac{\gamma^2}{2\delta} & k_b k_{bc} - \frac{\gamma^2}{2\delta} & k_b k_{cc} + \alpha - \frac{\gamma^2}{2\delta} & \beta - \frac{\gamma^2}{2\delta} \\ k_f k_{ad} - \frac{\gamma^2}{2\delta} & -\frac{\gamma^2}{2\delta} & \beta - \frac{\gamma^2}{2\delta} & k_f k_{dd} + \alpha - \frac{\gamma^2}{2\delta} \end{bmatrix} \begin{Bmatrix} \theta_a \\ \theta_b \\ \theta_c \\ \theta_d \end{Bmatrix} = \{\,\mathbf{0}\,\} \tag{7}$$

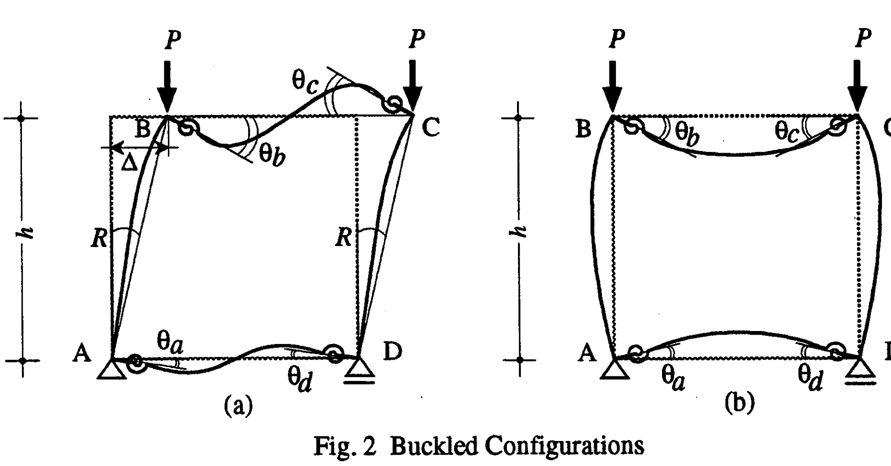


Fig. 2 Buckled Configurations

Buckling without sidesway:

$$\begin{bmatrix} k_f k_{aa}+\alpha & \beta & 0 & k_f k_{ad} \\ \beta & k_b k_{bb}+\alpha & k_b k_{bc} & 0 \\ 0 & k_b k_{bc} & k_b k_{cc}+\alpha & \beta \\ k_f k_{ad} & 0 & \beta & k_f k_{dd}+\alpha \end{bmatrix} \begin{Bmatrix} \theta_a \\ \theta_b \\ \theta_c \\ \theta_d \end{Bmatrix} = \{ \mathbf{0} \} \tag{8}$$

The load parameter Z_{cr} at which the frame buckles is obtained as the smallest value of Z which satisfies the condition that the determinant of the coefficient matrix of the simultaneous equations given by Eqs. (7) and (8) becomes zero.

2.3 Approximate Analysis

Approximate analysis proposed here replaces a given frame with beam springs shown in Fig. 3(a) by an equivalent rigid frame shown in Fig. 3(b), and then calculates the buckling strength from the column effective length determined from the conventional alignment charts. For simplicity, it is assumed that the spring constants of two springs in the beam are identical, as shown in Fig. 3(a). They may be different in a real case, but the method shown below is still applicable by assuming that the spring constant of the spring in the frame shown in Fig. 3(a) is given by the mean of two spring constants, i.e.,

$$k_{bs}=\frac{1}{2}(k_{bs1}+k_{bs2}), \qquad k_{fs}=\frac{1}{2}(k_{fs1}+k_{fs2}) \tag{9}$$

(i) Buckling with sidesway

Referring to Fig. 2(a), if the spring constants of two springs are identical in each of the beams BC and AD, the frame becomes symmetrical, and the deformation of sway buckling becomes anti-symmetrical, i. e., $\theta_b = \theta_c$. Substituting $\theta_b = \theta_c$ into Eq. (1) leads to

$$M_{bc} = k_b K_c (k_{bb} + k_{bc})\theta_b \tag{10}$$

where $(k_{bb} + k_{bc})$ is obtained from Eq. (2) in view of $k_{bs1} = k_{bs2} = k_{bs}$, as follows:

$$k_{bb} + k_{bc} = \frac{6\,k_{bs}}{k_{bs} + 6\,k_b} \tag{11}$$

On the other hand, beam-end moment M_{bc} of the equivalent rigid frame shown in Fig. 3(b) caused by the anti-symmetrical buckling with sidesway is expressed as

$$M_{bc} = 6\,k_b' K_c \theta_b \tag{12}$$

Equating M_{bc} given by Eqs. (10) and (12) leads to the expression for the beam stiffness ratio k_b' of the equivalent rigid frame, as follows:

$$k_b' = \frac{k_b k_{bs}}{k_{bs} + 6\,k_b} \tag{13}$$

Similarly,

$$k_f' = \frac{k_f k_{fs}}{k_{fs} + 6\,k_f} \tag{14}$$

Setting up the equilibrium conditions of bending moments at the panel points A and B of the frame shown in Fig. 3(b) which

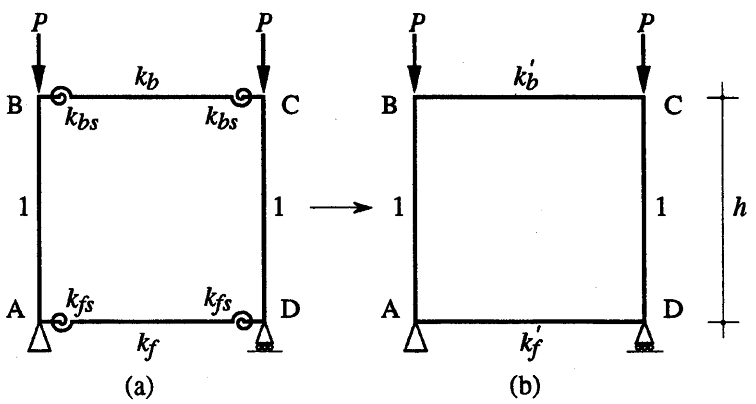


Fig. 3 Equivalent Rigid Frame

buckles in anti-symmetrical configuration with sidesway, and eliminating the chord rotation from the equilibrium condition of story shear, i. e., V_{ab} in Eq. (4) = 0, leads to a set of homogeneous simultaneous equations of size 2, which controls the anti-symmetrical buckling of the rigid frame shown in Fig. 3(b), as follows:

$$\begin{bmatrix} 6k_f' + \alpha - \frac{\gamma^2}{\delta} & \beta - \frac{\gamma^2}{\delta} \\ \beta - \frac{\gamma^2}{\delta} & 6k_b' + \alpha - \frac{\gamma^2}{\delta} \end{bmatrix} \begin{Bmatrix} \theta_a \\ \theta_b \end{Bmatrix} = \{ 0 \} \tag{15}$$

Equating the determinant of the coefficient matrix of Eq. (15) to zero, the following equation is obtained:

$$\left(6k_b' + \alpha - \frac{\gamma^2}{\delta}\right)\left(6k_f' + \alpha - \frac{\gamma^2}{\delta}\right) - \left(\beta - \frac{\gamma^2}{\delta}\right)^2 = 0 \tag{16}$$

The equation to determine the buckling strength Z_{cr} is obtained by substituting the expressions in Eq. (5) into Eq. (16), as follows:

$$\frac{Z_{cr}^2}{6(k_b' + k_f')} - \frac{6k_b' k_f'}{k_b' + k_f'} = \frac{Z_{cr}}{\tan Z_{cr}} \tag{17}$$

(ii) Buckling without sidesway

In the case of the symmetrical buckling without sidesway of the frame shown in Fig. 3(b), the buckling condition is obtained by a similar manipulation, noting that $\theta_b = -\theta_c$. Equations (18) through (24) below correspond to Eqs. (10) through (16), respectively. Equilibrium of story shear is not necessary in this case.

$$M_{bc} = k_b K_c (k_{bb} - k_{bc}) \theta_b \tag{18}$$

$$k_{bb} - k_{bc} = \frac{2k_{bs}}{k_{bs} + 2k_b} \tag{19}$$

$$M_{bc} = 2k_b' K_c \theta_b \tag{20}$$

$$k_b' = \frac{k_b k_{bs}}{k_{bs} + 2k_b} \tag{21}$$

$$k_f' = \frac{k_f k_{fs}}{k_{fs} + 2k_f} \tag{22}$$

$$\begin{bmatrix} 2k_f' + \alpha & \beta \\ \beta & 2k_b' + \alpha \end{bmatrix} \begin{Bmatrix} \theta_a \\ \theta_b \end{Bmatrix} = \{ 0 \} \tag{23}$$

$$(2k_b' + \alpha)(2k_f' + \alpha) - \beta^2 = 0 \tag{24}$$

Finally, the equation to determine the buckling strength Z_{cr} is obtained by substituting the expressions in Eq. (5) into Eq. (24), as follows:

$$\frac{Z_{cr}^2}{4k_b' k_f'} + \frac{\left(1 - \frac{Z_{cr}}{\tan Z_{cr}}\right)\left(k_b' + k_f'\right)}{2k_b' k_f'} + \frac{2\tan\frac{Z_{cr}}{2}}{Z_{cr}} = 1 \tag{25}$$

Note that Eqs. (17) and (25) are the basic equations from which conventional alignment charts to determine the effective length of framed columns[11].

2.4 Numerical Examples

In order to investigate the accuracy of the buckling strength determined by the approximate method proposed above, 3 types of semirigid frames shown in Fig. 4 have been analyzed: (a) column tops are semirigidly connected to the beams, and column bases are fixed, (b) column tops are rigidly connected to the beams, and column bases are semirigidly connected to the beams, and (c) columns are semirigidly connected to the beams at

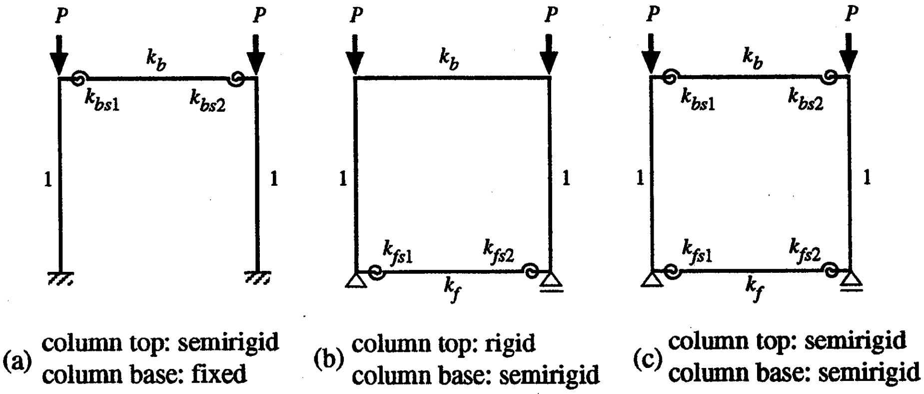


Fig. 4 Numerical Examples of Semirigid Rectangular Frames

both ends. The stiffness ratios of the beams and the springs are set in the following manner: i) upper beam stiffness ratio $k_b = 2$ or 5, ii) lower beam stiffness ratio $k_f = 10$ times k_b, iii) ratio of left and right spring constants $k_{bs1} : k_{bs2}$ and $k_{fs1} : k_{fs2} = 1 : 0.8$ or $1 : 0.5$, iv) ratio of left spring constant to beam stiffness $k_{bs1} : k_{bs}$ and $k_{fs1} : k_{fs} = 1.5$ or 0.5, and v) in addition, the case of k_b being changed from 2 to 5 with all other parameters being kept unchanged.

The buckling condition for the portal frame with the fixed column base cannot be deduced from the equations shown above, and thus they have been derived separately, and obtained as shown below:

i) Exact analysis

Buckling with sidesway:

$$\left(k_b k_{bb} + \alpha - \frac{\gamma^2}{\delta}\right)\left(k_b k_{cc} + \alpha - \frac{\gamma^2}{\delta}\right) - \left(k_b k_{bc} + \alpha - \frac{\gamma^2}{\delta}\right)^2 = 0 \tag{26}$$

Buckling without sidesway:

$$(k_b k_{bb} + \alpha)(k_b k_{cc} + \alpha) - (k_b k_{bc})^2 = 0 \tag{27}$$

ii) Approximate analysis

Buckling with sidesway:

$$6k_b' + \alpha - \frac{\gamma^2}{\delta} = 0 \tag{28}$$

Buckling without sidesway:

$$2k_b' + \alpha = 0 \tag{29}$$

Results of the numerical calculation of buckling strength are tabulated in Tables 1(a) through (c) corresponding to the frames shown in Figs. 4(a) through (c), respectively. In each table, "Error" indicates a ratio of the difference between the values of Z_{cr} obtained by the exact and approximate analyses to the value of the exact solution, and the negative value means that the approximate solution is higher than the exact solution. The maximum error in each group is underlined.

In addition to the numerical examples described above, the frames shown in Fig. 4 with extremely large spring constants, i. e., 100 or 1000 times the beam stiffness, are analyzed exactly, for the case of spring constants of two springs in a beam being identical, i. e., $k_{bs1} = k_{bs2} = k_{bs}$ and $k_{fs1} = k_{fs2} = k_{fs}$. These examples are to see the value of spring constants to treat the connection as rigid. Results are shown in Table 2, where "Error" indicates a ratio of the difference between the values of Z_{cr} obtained by the exact analysis treating the connection as rigid (k_{bs} and/or $k_{fs} = \infty$) and treating it as semirigid to the solution obtained for the rigid connection. The buckling conditions for the rigid frame are simply obtained from Eqs. (17) and (25) for rectangular frames, and from Eqs. (28) and (29) for portal frames with the fixed column base, by replacing k_b' and k_f' by k_b and k_f, respectively.

Table 1 Buckling Strength of Semirigid Rectangular Frames
(a) column top: semirigid, column base: fixed

k_b	k_{bs1}	k_{bs2}	Sidesway permitted			Sidesway prevented		
			Exact	Approx.	Error	Exact	Approx.	Error
2	3	2.4	2.326	2.329	-0.001	4.930	4.938	-0.002
2	3	1.5	2.245	2.267	-0.010	4.836	4.899	-0.013
5	7.5	6	2.685	2.688	-0.001	5.321	5.332	-0.002
5	7.5	3.75	2.608	2.634	-0.010	5.181	5.278	-0.019
2	1	0.8	1.971	1.972	-0.001	4.710	4.722	-0.002
2	1	0.5	1.914	1.923	-0.004	4.642	4.692	-0.011
5	2.5	2	2.305	2.307	-0.001	4.964	4.985	-0.004
5	2.5	1.25	2.220	2.238	-0.008	4.831	4.931	-0.021
5	3	2.4	2.374	2.377	-0.001	5.022	5.043	-0.004
5	3	1.5	2.287	2.307	-0.009	4.880	4.985	-0.022
5	1	0.8	1.984	1.985	0.000	4.728	4.746	-0.004
5	1	0.5	1.926	1.933	-0.004	4.649	4.711	-0.013

Table 1 Buckling Strength of Semirigid Rectangular Frames (continued)
(b) column top: rigid, column base: semirigid

k_b	k_f	k_{fs1}	k_{fs2}	Sidesway permitted			Sidesway prevented		
				Exact	Approx.	Error	Exact	Approx.	Error
2	20	30	24	2.786	2.788	0.000	5.056	5.058	0.000
2	20	30	15	2.758	2.770	-0.004	5.004	5.030	-0.005
5	50	75	60	2.988	2.988	0.000	5.628	5.629	0.000
5	50	75	37.5	2.974	2.980	-0.002	5.601	5.615	-0.002
2	20	10	8	2.621	2.623	-0.001	4.813	4.818	-0.001
2	20	10	5	2.556	2.580	-0.009	4.705	4.761	-0.012
5	50	25	20	2.906	2.907	0.000	5.486	5.490	-0.001
5	50	25	12.5	2.869	2.884	-0.005	5.405	5.451	-0.009
5	50	30	24	2.926	2.927	0.000	5.520	5.523	-0.001
5	50	30	15	2.894	2.907	-0.005	5.451	5.490	-0.007
5	50	10	8	2.743	2.746	-0.001	5.226	5.235	-0.002
5	50	10	5	2.674	2.699	-0.009	5.079	5.168	-0.017

Following characteristics are observed from the results shown in Table 1: i) Unsafe estimates are obtained from the approximate analysis in all cases calculated. ii) Error obtained for the buckling with sidesway is larger than that for the buckling without sidesway, but the reason is not yet known. iii) Most of the examples with the ratio of two spring constants equal to 1 : 0.8 show the error less than 1%, and the accuracy of the approximate solution is very good. iv) The maximum error among the examples with the ratio of two spring constants equal to 1 : 0.5 is 4.6%, which occurs in the frame with semirigid connections at both top and base of the column, shown in Fig. 4(c), but it is sufficiently small. It is observed from Table 2 that the semirigid connection can be treated as rigid, if the spring constant is larger than 100 times the beam stiffness.

Table 1 Buckling Strength of Semirigid Rectangular Frames (continued)
(c) column top: semirigid, column base: semirigid

k_b	k_f	k_{bs1}	k_{bs2}	k_{fs1}	k_{fs2}	Sidesway permitted			Sidesway prevented		
						Exact	Approx.	Error	Exact	Approx.	Error
2	20	3	2.4	30	24	2.242	2.245	-0.001	4.672	4.685	-0.003
2	20	3	1.5	30	15	2.147	2.174	-0.012	4.509	4.622	-0.025
5	50	7.5	6	75	60	2.643	2.646	-0.001	5.203	5.217	-0.003
5	50	7.5	3.75	75	37.5	2.559	2.587	-0.011	5.029	5.150	-0.024
2	20	1	0.8	10	8	1.794	1.796	-0.001	4.229	4.259	-0.007
2	20	1	0.5	10	5	1.703	1.721	-0.011	4.020	4.180	-0.040
5	50	2.5	2	25	20	2.218	2.221	-0.001	4.726	4.758	-0.007
5	50	2.5	1.25	25	12.5	2.117	2.140	-0.011	4.498	4.673	-0.039
5	50	3	2.4	30	24	2.298	2.301	-0.001	4.813	4.842	-0.006
5	50	3	1.5	30	15	2.196	2.221	-0.011	4.589	4.758	-0.037
5	50	1	0.8	10	8	1.813	1.815	-0.001	4.275	4.321	-0.011
5	50	1	0.5	10	5	1.720	1.737	-0.010	4.045	4.232	-0.046

Table 2 Buckling Strength of Semirigid Rectangular Frames
- Effect of Stiffness of Springs-
(a) column top: semirigid, column base: fixed

k_b	k_{bs}	Sidesway permitted		Sidesway prevented	
		Z_{cr}	Error	Z_{cr}	Error
2	10	2.684	0.076	5.170	0.030
2	20	2.786	0.041	5.241	0.016
2	200	2.892	0.004	5.319	0.002
2	2000	2.903	0.000	5.328	0.000
2	∞	2.904	-	5.329	-
5	25	2.930	0.036	5.610	0.026
5	50	2.984	0.019	5.680	0.014
5	500	3.035	0.002	5.750	0.001
5	5000	3.040	0.000	5.757	0.000
5	∞	3.041	-	5.758	-

Table 2 Buckling Strength of Semirigid Rectangular Frames
- Effect of Stiffness of Springs- (continued)
(b) column top: rigid, column base: semirigid

k_b	k_f	k_{fs}	Sidesway permitted		Sidesway prevented	
			Z_{cr}	Error	Z_{cr}	Error
2	20	100	2.856	0.009	5.168	0.008
2	20	200	2.869	0.005	5.190	0.004
2	20	2000	2.881	0.000	5.210	0.000
2	20	20000	2.882	0.000	5.212	0.000
2	20	∞	2.882	-	5.212	-
5	50	250	3.019	0.004	5.684	0.004
5	50	500	3.025	0.002	5.695	0.002
5	50	5000	3.030	0.000	5.704	0.000
5	50	50000	3.031	0.000	5.705	0.000
5	50	∞	3.031	-	5.705	-

Table 2 Buckling Strength of Semirigid Rectangular Frames
- Effect of Stiffness of Springs- (continued)
(c) column top: semirigid, column base: semirigid

k_b	k_f	k_{bs}	k_{fs}	Sidesway permitted		Sidesway prevented	
				Z_{cr}	Error	Z_{cr}	Error
2	20	10	100	2.856	0.009	5.014	0.038
2	20	20	200	2.869	0.005	5.105	0.021
2	20	200	2000	2.868	0.005	5.201	0.002
2	20	2000	20000	2.881	0.000	5.211	0.000
2	20	∞	∞	2.882	-	5.212	-
5	50	25	250	2.910	0.040	5.539	0.029
5	50	50	500	2.969	0.020	5.618	0.015
5	50	500	5000	3.024	0.002	5.696	0.002
5	50	5000	50000	3.030	0.000	5.704	0.000
5	50	∞	∞	3.031	-	5.705	-

3. Elastic Buckling Analysis of Semirigid Fish-Bone Frames

3.1 Model for the Analysis

Figure 5(a) shows a model for the analysis, which is a fish-bone frame with linear springs in each beam at near end to the column, subjected to vertical load P on the column. Far end of each beam is simply supported. Stiffness of two beams in the same floor is identical. Stiffness ratios of the column in i-th story, the beam and two springs in i-th floor are denoted by k_{ci}, k_{bi}, k_{sai} and k_{sbi}, respectively. Column height is identical for all columns. This chapter is to investigate the applicability of conventional alignment charts to determine the buckling strength of a column in a semirigid multistory frame.

3.2 Exact Analysis

The slope-deflection formulas for the member-end moments at i-th panel point of two beams, M_{ciai} and M_{cibi}, two columns, $M_{cici\text{-}1}$ and M_{cici+1}, and the lateral force V_{cici+1} in i-th column are expressed as follows:

$$M_{c_ia_i} = k_{bi} K_0 \frac{3k_{sai}}{k_{sai} + 3k_{bi}} \theta_i$$
$$M_{c_ib_i} = k_{bi} K_0 \frac{3k_{sbi}}{k_{sbi} + 3k_{bi}} \theta_i \tag{30}$$

$$M_{c_ic_{i-1}} = k_{ci\text{-}1} K_0 (\alpha_{i-1}\theta_i + \beta_{i-1}\theta_{i-1} - \gamma_{i-1}R_{i-1})$$
$$M_{c_ic_{i+1}} = k_{ci} K_0 (\alpha_i\theta_i + \beta_i\theta_{i+1} - \gamma_i R_i) \tag{31}$$

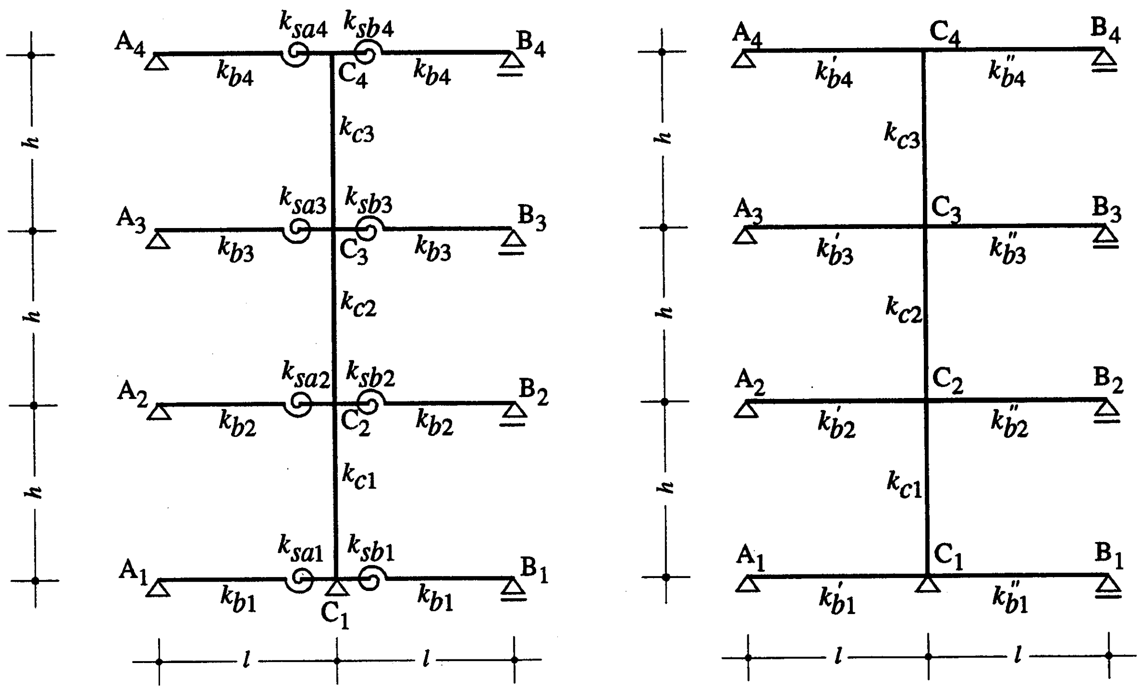
$$V_{c_ic_{i+1}} = -\frac{k_{ci}}{h} K_0 (\gamma_i\theta_i + \gamma_i\theta_{i+1} - \delta_i R_i) \tag{32}$$

where K_0 denotes the stiffness of the reference member. Coefficients appearing in Eq. (30) are derived from k_{bb} in Eq. (2) by substituting $k_{bs2} = 0$, i. e., the rotation is free at point B_i. Stability functions appearing in Eqs. (31) and (32) are obtained by Eq. (5) with the load parameter given by

$$Z_i = h\sqrt{\frac{N_i}{EI_i}} \tag{33}$$

where N_i and I_i denote the axial force and the moment of inertia of the column in i-th story.

Figure 6(a) shows the deformations occurring when the frame buckles with sidesway. In this case, equilibrium of bending moments acting around 4 panel points C_1 through C_4, and equilibrium of story shear in 3 stories lead to 7 equations, and eliminating the chord rotation R_i leads to a set of homogeneous simultaneous equations of size 4, with the unknowns θ_1 through θ_4, shown below. In the case of the buckling without sidesway shown in Fig. 6(b), the equilibrium of story shear is not necessary, since the chord rotation does not appear, and a similar set of homogeneous simultaneous equations is obtained.



(a) Semirigid Fish-Bone Frame (b) Equivalent Rigid Fish-Bone Frame

Fig. 5 Model for Analysis: Semirigid Fish-Bone Frame

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{Bmatrix} = \{ 0 \} \tag{34}$$

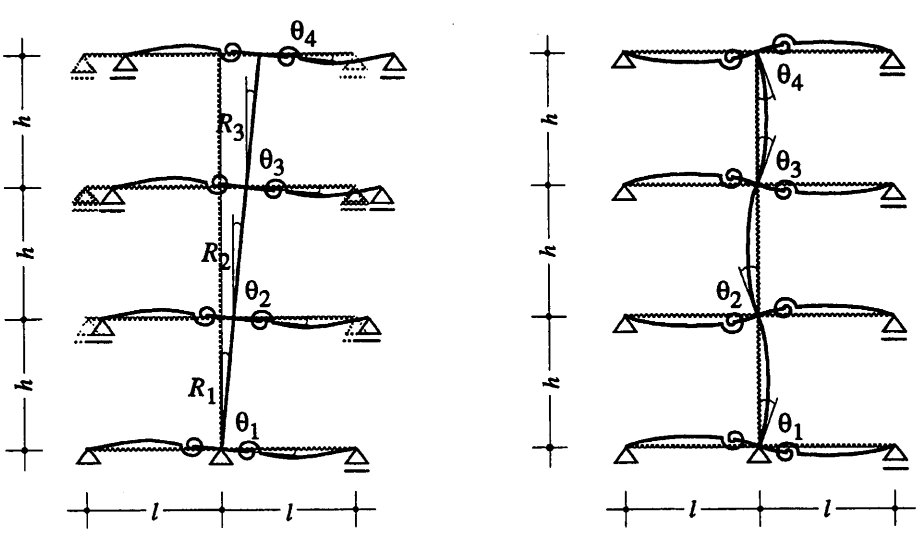
Derivation of each element in the coefficient matrix in Eq. (34) is omitted here, and only the final forms are shown below.

Buckling with sidesway:

$$\begin{aligned}
A_{11} &= k_{c1}\left(\alpha_1 - \frac{\gamma_1^2}{\delta_1}\right) + 3k_{b1}\left(\frac{k_{sa1}}{k_{sa1}+3k_{b1}} + \frac{k_{sb1}}{k_{sb1}+3k_{b1}}\right) \\
A_{22} &= k_{c1}\left(\alpha_1 - \frac{\gamma_1^2}{\delta_1}\right) + k_{c2}\left(\alpha_2 - \frac{\gamma_2^2}{\delta_2}\right) + 3k_{b2}\left(\frac{k_{sa2}}{k_{sa2}+3k_{b2}} + \frac{k_{sb2}}{k_{sb2}+3k_{b2}}\right) \\
A_{33} &= k_{c2}\left(\alpha_2 - \frac{\gamma_2^2}{\delta_2}\right) + k_{c3}\left(\alpha_3 - \frac{\gamma_3^2}{\delta_3}\right) + 3k_{b3}\left(\frac{k_{sa3}}{k_{sa3}+3k_{b3}} + \frac{k_{sb3}}{k_{sb3}+3k_{b3}}\right) \\
A_{44} &= k_{c3}\left(\alpha_3 - \frac{\gamma_3^2}{\delta_3}\right) + 3k_{b4}\left(\frac{k_{sa4}}{k_{sa4}+3k_{b4}} \cdot \frac{k_{sb4}}{k_{sb4}+3k_{b4}}\right) \\
A_{12} &= A_{21} = k_{c1}\left(\beta_1 - \frac{\gamma_1^2}{\delta_1}\right) \qquad A_{34} = A_{43} = k_{c3}\left(\beta_3 - \frac{\gamma_3^2}{\delta_3}\right) \\
A_{23} &= A_{32} = k_{c2}\left(\beta_2 - \frac{\gamma_2^2}{\delta_2}\right) \qquad A_{13} = A_{31} = A_{14} = A_{41} = A_{24} = A_{42} = 0
\end{aligned} \tag{35}$$

Buckling without sidesway:

$$\begin{aligned}
A_{11} &= k_{c1}\alpha_1 + 3k_{b1}\left(\frac{k_{sa1}}{k_{sa1}+3k_{b1}} \cdot \frac{k_{sb1}}{k_{sb1}+3k_{b1}}\right) \\
A_{22} &= k_{c1}\alpha_1 + k_{c2}\alpha_2 + 3k_{b2}\left(\frac{k_{sa2}}{k_{sa2}+3k_{b2}} + \frac{k_{sb2}}{k_{sb2}+3k_{b2}}\right) \\
A_{33} &= k_{c2}\alpha_2 + k_{c3}\alpha_3 + 3k_{b3}\left(\frac{k_{sa3}}{k_{sa3}+3k_{b3}} + \frac{k_{sb3}}{k_{sb3}+3k_{b3}}\right)
\end{aligned} \tag{36}$$



(a) (b)

Fig. 6 Buckled Configuration

$$A_{44} = k_{c3}\,\alpha_3 + 3\,k_{b4}\left(\frac{k_{sa4}}{k_{sa4}+3k_{b4}}\cdot\frac{k_{sb4}}{k_{sb4}+3\,k_{b4}}\right)$$

$$A_{12} = A_{21} = k_{c1}\beta_1 \qquad A_{34} = A_{43} = k_{c3}\,\beta_3 \tag{36}$$

$$A_{23} = A_{32} = k_{c2}\,\beta_2 \qquad A_{13} = A_{31} = A_{14} = A_{41} = A_{24} = A_{42} = 0$$

The load parameter Z_{cr} at which the frame buckles is obtained as the smallest value of Z which satisfies the condition that the determinant of the coefficient matrix of the simultaneous equations given by Eq. (34) becomes zero.

3.3 Approximate Analysis

As done in Section 2.3, first consider a rigid fish-bone multistory frame shown in Fig. 5(b) which is equivalent to the semirigid frame shown in Fig. 5(a). The expression of end moment of a beam whose far end is simply supported is given as follows:

$$M_{c_ia_i} = 3k_{bi}'\,K_0\,\theta_a\,, \qquad M_{c_ib_i} = 3k_{bi}''\,K_0\,\theta_a \tag{37}$$

Equating M_{ciai} and M_{cibi} given by Eqs. (30) and (37) leads to the expression for the beam stiffness ratio k_b' and k_b'' of the equivalent rigid frame, as follows:

$$k_{bi}\frac{3\,k_{sai}}{k_{sai}+3k_{bi}} = 3\,k_{bi}' \qquad \therefore\ k_{bi}' = \frac{k_{bi}\,k_{sai}}{k_{sai}+3k_{bi}}$$

$$k_{bi}\frac{3\,k_{sbi}}{k_{sbi}+3k_{bi}} = 3\,k_{bi}'' \qquad \therefore\ k_{bi}'' = \frac{k_{bi}\,k_{sbi}}{k_{sbi}+3k_{bi}} \tag{38}$$

The effective length factor γ_i of each column in the rigid fish-bone frame is evaluated from the alignment charts of Ref. [11], using following G-factor evaluated at the panel points C_1 through C_4:

Buckling with sidesway:

$$G_{C_i} = \frac{2(k_{ci\text{-}1} + k_{ci})}{k_{bi}' + k_{bi}''} \tag{39}$$

Buckling without sidesway:

$$G_{C_i} = \frac{2(k_{ci\text{-}1} + k_{ci})}{3\,(k_{bi}' + k_{bi}'')} \tag{40}$$

Finally, the value of the load parameter Zcri when the column in i-th story buckles is given as follows:

$$P_{cri} = \frac{\pi^2 EI_i}{(\gamma_i h)^2}\ ; \qquad Z_{cri} = h\sqrt{\frac{P_{cri}}{EI_i}} = \frac{\pi}{\gamma_i} \tag{41}$$

3.4 Numerical Examples

In order to investigate the accuracy of the buckling strength determined by applying the alignment charts to the semirigid frames, 3 semirigid fish-bone frames shown in Fig. 7 have been analyzed: The frame (a) simulates 3 stories at the top of the high-rise building frame, in which the column stiffness increases in the lower stories, while the frame (b) simulates bottom 3 stories, in which the column stiffness is constantly distributed. In these two frames, the beam and spring stiffness are constantly distributed. The frame (c) corresponds to the frame (a), but the beam and spring stiffness increase in the lower floor level. Axial forces in each column are distributed along the height proportional to the distribution of the column stiffness, and therefore the value of the load parameter Z_i becomes identical in each story, according to Eq. (33).

Results of the numerical calculation of buckling strength are tabulated in Table 3, where (a) through (c) correspond to the frames (a) through (c) shown in Fig. 7, respectively. Z_{cr} indicates the critical value of Z obtained from the exact analysis, while Z_{cri} is the value determined from the alignment charts for the column in each story.

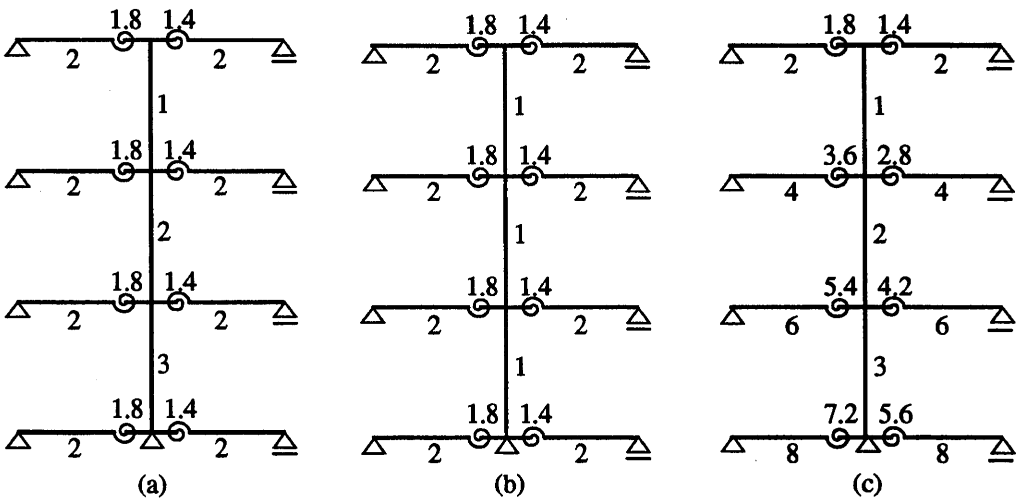


Fig. 7 Numerical Examples of Semirigid Fish-Bone Frames

Table 3 Buckling Strength of Semirigid Fish-Bone Frames

	(a)		(b)		(c)	
	Sidesway permitted	Sidesway prevented	Sidesway permitted	Sidesway prevented	Sidesway permitted	Sidesway prevented
Z_{cr}	1.481	3.567	1.778	3.966	1.907	4.112
Z_{cr1}	1.237 (0.164)	3.516 (0.014)	1.869 (<u>-0.051</u>)	3.998 (-0.008)	1.972 (-0.034)	4.154 (-0.010)
Z_{cr2}	1.237 (0.164)	3.516 (0.014)	1.707 (0.040)	3.784 (<u>0.046</u>)	1.829 (<u>0.041</u>)	3.915 (<u>0.048</u>)
Z_{cr3}	1.758 (<u>-0.187</u>)	3.903 (<u>-0.094</u>)	1.869 (<u>-0.051</u>)	3.998 (-0.008)	1.947 (-0.021)	4.081 (0.008)

"Error" indicates a ratio of the difference between the values of Z_{cr} and Z_{cri} to the value of Z_{cr}, and the negative value means that the alignment chart solution is higher than the exact solution. The maximum error in each group is underlined.

Following characteristics are observed from the results shown in Table 3: i) Safe and unsafe estimates obtained from the alignment charts are mixedly appear, and no tendency is found. ii) Error obtained for the buckling with sidesway is larger than that for the buckling without sidesway except for a few cases, but the reason is not yet known. iii) The maximum error for the frame (a) is 18.7% in the unsafe side, and the accuracy of the alignment chart solution for the frame (a) is rather bad. On the other hand, the accuracy for the frames (b) and (c) is very good, with the maximum error is 5.1%. This is understandable, since the alignment chart is designed to apply to regular multistory frames in which member stuffiness and axial forces in columns are distributed in a good balance.

4. Conclusions

i) Approximate method has been proposed for the analysis of the buckling strength of semirigid frames, which replaces semirigid frames by equivalent rigid frames and utilizes the conventional alignment charts to evaluate the column effective length.

ii) Numerical examples of semirigid rectangular frames has shown that the accuracy of the approximate solution is very good, unless the difference in spring constants of two springs in a beam is too large.

iii) Numerical examples of semirigid rectangular frames has shown that the semirigid connection can be treated as

rigid, if the spring constant is larger than 100 times the beam stiffness.
iv) Numerical examples of semirigid fish-bone frames has shown that the accuracy of the alignment chart solution is very good for the frames in which member and spring stuffiness and axial forces in columns are distributed in a good balance. Otherwise, it becomes bad, since the chart is not designed for the application to unbalanced and irregular frames.
v) It must be noted that the approximate method has given unsafe estimate to the buckling strength in most of the numerical examples.

References

[1] Jones, S. W., Kirby, P. A. and Nethercot, D. A.: Columns with Semirigid Joints, Journal of the Structural Division, Proceedings of American Society for Civil Engineers (ASCE), Vol. 108, No. 2 pp. 361-372, 1982.2.

[2] Barakat, M. A. and Chen, W. F.: Practical Analysis of Semi-rigid Frames, Engineering Journal, American Institute of Steel Construction (AISC), Vol. 27, No. 2, pp. 54-68, 1990.

[3] Kishi, N., Goto, Y., Chen, W. F. and Komuro, M.: An Elastic Method of Column Effective Length Factor in Semi-Rigid Braced Frames, Journal of Constructional Steel, Vol. 3, pp. 53-60, 1995.11. (in Japanese)

[4] Kishi, N., Goto, Y., Chen, W. F. and Komuro, M.: An Esimation Method of Column Effective Length Factor in Semi-Rigid Sway Frames, Journal of Structural Engineering, Capan Society for Civil Engineers and AIJ, Vol. 41A, pp. 153-161, 1995.3. (in Japanese)

[5] Kishi, N., Chen, W. F. and Goto, Y.: Effective Length Factor of Columns in Semirigid and Unbraced Frames, Journal of Structural Engineering, ASCE, Vol. 123, No. 3, pp. 313-320, 1997.3.

[6] Chen, W. F. and Kishi, N.: Semi-rigid Steel Beam-to-Column Connections: Data Base and Modeling, Journal of Structural Engineering, ASCE, Vol. 115, No. 1, pp. 105-119, 1989.1.

[7] Nishiura, H., Ohtani, Y., Ohtani, K. and Tsuji, B.: Effect of Strength and Stiffness of Semi-Rigid Connection on Frame Stability, Summary of Technical Reports, Annual Meeting of Architectural Institute of Japan (AIJ), Structure III, pp. 299-300, 1995.8. (in Japanese)

[8] Matsui, C., Kawano, A. and Ukishima, T.: Influence of Partial Base Fixity on the Effective Length of Column Members in Steel Frames, Summary of Technical Reports, Annual Meeting of AIJ, Structure III, pp. 873-874, 1986.8. (in Japanese)

[9] Aristizabal-Ochoa, J. D.: First- and Second-Order Stiffness Matrices and Load Vector of Beam-Columns with Semirigid Connection, Journal of Structural Engineering, ASCE, Vol. 123, No. 5, pp. 669-678, 1997.5.

[10] Barakat, M. A. and Chen, W. F.: Design Analysis of Semi-rigid Frames, Evaluation and Implementation, Engineering Journal, AISC, Vol. 28, No. 2, pp. 55-64, 1991.

[11] Recommendations for the Plastic Design of Steel Structures, AIJ, 1975.9. (in Japanese)

[12] Advanced Theory of Steel Structures, Edited by M. Wakabayashi, Maruzen Co. Ltd., 1985.5. (in Japanese)