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On the Symbolization of I-proposition

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In this note, we consider the symbolization of **I**-proposition under the existential import, in an informal way.

In modern logic, the so-called **A**, **E**, **I** and **O**-propositions are generally symbolized respectively as $\forall x[F(x) \supset G(x)]$, $\forall x[F(x) \supset \sim G(x)]$, $\exists x[F(x) \wedge G(x)]$ and $\exists x[F(x) \wedge \sim G(x)]$. And in this symbolization, by adding the existential import, we can easily show that four oppositions (in traditional logic) hold.

Now, each of the following ten formulas obviously holds:

(i) $\forall x[F(x) \supset G(x)] \supset \exists x[F(x) \supset G(x)]$,
$\forall x \sim [F(x) \supset G(x)] \supset \exists x \sim [F(x) \supset G(x)]$,
$\sim \exists x[F(x) \supset G(x)] \supset \sim \forall x[F(x) \supset G(x)]$,
$\sim \exists x \sim [F(x) \supset G(x)] \supset \sim \forall x \sim [F(x) \supset G(x)]$.

(ii) $\forall x \sim [F(x) \supset G(x)] \equiv \sim \exists x[F(x) \supset G(x)]$,
$\sim \forall x \sim [F(x) \supset G(x)] \equiv \exists x [F(x) \supset G(x)]$.

(iii) $\forall x[F(x) \supset G(x)] \supset \sim \forall x \sim [F(x) \supset G(x)]$,
$\forall x \sim [F(x) \supset G(x)] \supset \sim \forall x[F(x) \supset G(x)]$.

(iv) $\sim \exists x[F(x) \supset G(x)] \supset \exists x \sim [F(x) \supset G(x)]$,
$\sim \exists x \sim [F(x) \supset G(x)] \supset \exists x[F(x) \supset G(x)]$.

It is seen from this that, if **E** is symbolized as $\forall x \sim [F(x) \supset G(x)]$ and **I** as $\exists x[F(x) \supset G(x)]$, then, without the existential import, four oppositions hold. (i, e., (i), (ii), (iii) and (iv) mean respectively subalternation, contradictory, contrary and subcontrary.) Furthermore, a simple argument shows that each of the formulas which are obtained by adding $\exists xF(x) \wedge \exists xG(x)$ to the antecedent of each of the above ten fomulas holds. That is, when **E** is interpreted as $\forall x \sim [F(x) \supset G(x)]$ and **I** as $\exists x[F(x) \supset G(x)]$, we can see that four oppositions hold even under the existential import. Thus, in view of four oppositions, it may seem that there is no difference between two symbolizations of **E** (or, **I**), i. e., $\forall x[F(x) \supset \sim G(x)]$ and $\forall x \sim [F(x) \supset G(x)]$ (or, $\exists x[F(x) \wedge G(x)]$ and $\exists x[F(x) \supset G(x)]$). Is there, however, no problem in saying that under the existential import, we can symbolize **E, I** as $\forall x \sim [F(x) \supset G(x)]$, $\exists x[F(x) \supset G(x)]$ respectively ?

We here notice that under the assumption of symbolization of **E** as $\forall x \sim [F(x) \supset G(x)]$, we can easily derive the symbolization of **I** as $\exists x[F(x) \supset G(x)]$ and *vice versa*, and that the same applies to the relation between the symbolization of **E** as $\forall x[F(x) \supset \sim G(x)]$ and that of **I** as $\exists x[F(x) \wedge G(x)]$. This means that there is a sort of provable equivalence between the symbolization of **E** as $\forall x \sim [F(x) \supset G(x)]$ (or, $\forall x [F(x) \supset \sim G(x)]$) and that of **I** as $\exists x [F(x) \supset G(x)]$ (or, $\exists x [F(x) \wedge G(x)]$). Hence, we may say that the above question reduces to the question : Is there no problem in saying that under the existential import, we can symbolize **I** as $\exists x [F(x) \supset G(x)]$?

As an example of **I**, let us take up the following proposition :

(α) Some girl is pretty.

Now what is meant by this proposition ? This means, it is thought, that there is (at least) one child who is classified as a girl, and that the girl has the property of being pretty. We here symbolize the propositional function 'x is a girl' as $G(x)$, and 'x is pretty' as $P(x)$. And let c be a child who satisfies the above proposition (α). (The existence of c is warranted by the existential import.) Then, it is easily seen that c has not only the property of being a girl, but that of being pretty. In other words, both $G(c)$ and $P(c)$, i. e., the formula '$G(c) \wedge P(c)$' holds. Therefore, using variable x, we could express the proposition (α) as $\exists x [G(x) \wedge P(x)]$. And, as was pointed out above, the propositon (α) obviously asserts that (at least) one child has above two properties. In case the child is a boy, however, it can not certainly be determined whether the child has the property of being pretty, or not. That is, (α) is to be interpreted as saying only that some girl is pretty, except what can be logically derived from it.

On the other hand, let us assume that (α) can be expressed as $\exists x [G(x) \supset P(x)]$. Then, for c above characterized, we can express the propoisition 'a girl c is pretty' as $G(c) \supset P(c)$. And again, we symbolize the propositional function 'x is a boy' as $B(x)$. Then, $B(c)$ is clearly false, and hence $B(c) \supset P(c)$ is true. Thus, under the premiss of $G(c) \supset P(c)$, we can assert $B(c) \supset P(c)$, since, in this case, under arbitrary premisses, we can assert $B(c) \supset P(c)$. (These arguments are based upon the well-known facts in elementary logic.) By the assumption concerning the expression of (α), this means that under the assumption that a girl c is pretty, we can assert that if c is a boy, then c is pretty. As we have noted, however, (α) is just the proposition concerning a girl c, and does not logically imply the proposition concerning a boy c.

Hence, from what has been stated above, we conclude that, under the existential import, **I** is to be symbolized as $\exists x [F(x) \wedge G(x)]$, rather than as $\exists x [F(x) \supset G(x)]$.

REFERENCES

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