

Original Paper

## Control of Robot Manipulators Using Disturbance Observer

Satoshi KOMADA, Muneaki ISHIDA, and Takamasa HORI  
(Department of Electrical and Electronic Engineering)

(Received September 20, 1994)

### *Abstract*

This paper proposes a simple and high performance position control method based on acceleration control. In conventional methods of disturbance compensation, it is difficult to realize the strict acceleration controller. Recently, a research on a disturbance observer shows that the disturbance compensation by the disturbance observer realizes the acceleration controller. Compared with the method of using inverse dynamics, the disturbance observer is simple and robust against parameter variation. The control algorithm is applied to a three-degree-of-freedom robot to show the effectiveness.

### *Key words*

Robot manipulators, Disturbance observer, Acceleration control, Inverse dynamics

## 1 Introduction

In order to produce high quality and low cost products, high speed and precise industrial manufacturing machines are necessary. For example, precise and high speed robot manipulators are required to perform many tasks in a short period. Movement of robot manipulators, however, are influenced by various parameter variations and the disturbances such as inertia force, gravity force, Coriolis force, centrifugal force, and friction force. Therefore, the high speed and precise control is deteriorated by the parameter variations and the disturbances, because the influence of them becomes large when robot manipulators move fast.

In conventional control of robot manipulators, inverse dynamics has been used to compensate the nonlinearity of robot manipulators[1][2], but it requires much computation time and is not robust against unknown disturbances. On the other hand, a disturbance observer which estimates and compensates the disturbances and the parameter variations of robot manipulators has been proposed[3][4][5]. The disturbance observer in joint space realizes acceleration control in joint space. This method requires less computation effort and has more robustness against the disturbance and the parameter variations than the conventional control methods.

In this paper, a robust position control using acceleration controller is presented. The proposed method is based on the acceleration in order to realize robust and fast response position control. To realize this system, the disturbance observer is used. The disturbance observer compensates the disturbances and the parameter variations of robot manipulators. The proposed method requires a little computation effort and has robustness against the disturbances and the parameter variations of robot manipulators. Experiments are performed by a robot manipulator to show the effectiveness.

## 2 System Linearization by Disturbance Observer in Joint Space

### 2.1 Definition of Disturbance in Joint Space

The motion equation of robot manipulators in joint space is given by

$$\mathbf{J}(\mathbf{q}(t))\ddot{\mathbf{q}}(t) + \mathbf{h}(\mathbf{q}(t), \dot{\mathbf{q}}(t)) + \mathbf{g}(\mathbf{q}(t)) + \mathbf{D}(t)\dot{\mathbf{q}}(t) + \mathbf{c}(t) + \mathbf{r}(\mathbf{q}(t), \mathbf{f}(t)) = \mathbf{K}_i(t)\mathbf{i}_q(t). \quad (1)$$

The motion equation is also illustrated as shown in Fig.1. The inertia matrix  $\mathbf{J}(\mathbf{q}(t))$ , the Coriolis and centrifugal force vector  $\mathbf{h}(\mathbf{q}(t), \dot{\mathbf{q}}(t))$ , and the gravity force vector  $\mathbf{g}(\mathbf{q}(t))$  are varied by grasped objects and manipulator's configuration.  $\mathbf{D}(t)$  is a diagonal viscosity matrix and  $\mathbf{c}(t)$  is a friction force vector. The reactive force vector from environments  $\mathbf{r}(\mathbf{q}(t), \mathbf{f}(t))$  arises when performing contact tasks.  $\mathbf{K}_i(t)$  is a diagonal torque constant matrix.

The proposed control method is required to realize an acceleration control system. As shown in (1) and Fig.1, it is difficult to control the joint acceleration  $\ddot{\mathbf{q}}(t)$  by  $\mathbf{i}_q(t)$  because of the disturbances and the parameter variations. Here, we introduce a linear system which can control acceleration. The linear system consists of a nominal torque diagonal matrix  $\mathbf{K}_{tn}$  and a nominal inertia diagonal matrix  $\mathbf{J}_n$  as shown in Fig.2 and (2).

$$\mathbf{K}_{tn}\mathbf{i}_q(t) = \mathbf{J}_n\ddot{\mathbf{q}}(t) \quad (2)$$

Where, the subscript  $n$  denotes the nominal parameter. Since  $\mathbf{K}_{tn}$  and  $\mathbf{J}_n$  are constant diagonal matrices without interference, the joint acceleration  $\ddot{\mathbf{q}}(t)$  of the linear system can be controlled by  $\mathbf{i}_q(t)$  arbitrary.

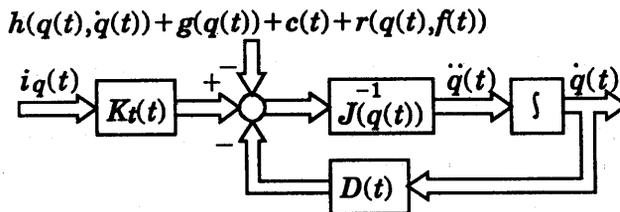


Fig.1: Block diagram of robot manipulator in joint space.

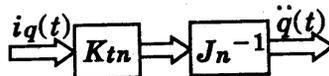


Fig.2: Linear system of robot manipulator in joint space.

Since there are many differences between (1) and (2), we have to compensate the differences in order to realize the linear system. The block diagram in joint space shown

in Fig.1 can be modified to that in Fig.3 by using the linear system in Fig.2. Here, by defining disturbance torque  $\tau_{dis}(t)$  as a difference between the real system and the linear system, the motion equation of robot manipulators in joint space is expressed as follows and its block diagram is shown in Fig.4.

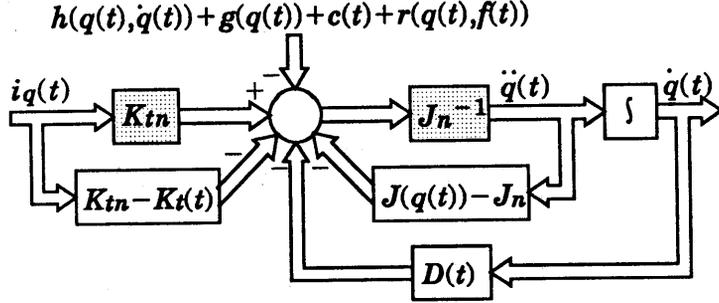


Fig.3: Block diagram in joint space using ideal linear system.

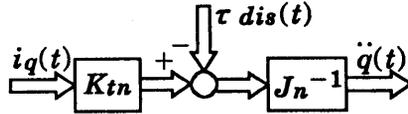


Fig.4: Block diagram in joint space based on definition of disturbance.

$$\mathbf{K}_{tn} \dot{\mathbf{q}}(t) = \mathbf{J}_n \ddot{\mathbf{q}}(t) + \boldsymbol{\tau}_{dis}(t) \quad (3)$$

Here, the joint disturbance torque vector  $\boldsymbol{\tau}_{dis}(t)$  is obtained from (1) and (3) as follows:

$$\begin{aligned} \boldsymbol{\tau}_{dis}(t) = & (\mathbf{J}(q(t)) - \mathbf{J}_n) \ddot{\mathbf{q}}(t) + \mathbf{h}(q(t), \dot{q}(t)) + \mathbf{g}(q(t)) \\ & + \mathbf{D}(t) \dot{q}(t) + \mathbf{c}(t) + \mathbf{r}(q(t), f(t)) + (\mathbf{K}_{tn} - \mathbf{K}_t(t)) \dot{\mathbf{q}}(t) \end{aligned} \quad (4)$$

The first term of the right hand side represents the inertia variation torque, the last term represents the torque ripple. If we compensate the disturbance vector, the linear system shown in Fig.2 is realized.

## 2.2 System Linearization by Disturbance Observer

Let  $\boldsymbol{\tau}_{dis}(s)$  denotes the Laplace transformation of the disturbance vector  $\boldsymbol{\tau}_{dis}(t)$ . In order to compensate the disturbance vector, we construct the disturbance observer which estimates the disturbance through a diagonal matrix of complementary sensitivity functions  $\mathbf{G}(s)$  as shown in (5) and Fig.5.

$$\hat{\boldsymbol{\tau}}_{dis}(s) = \mathbf{G}(s) \boldsymbol{\tau}_{dis}(s) \quad (5)$$

In Fig.5,  $\mathbf{S}$  represents a diagonal matrix having Laplace operators ( $diag(s, \dots, s)$ ). The system shown in Fig.5 is equivalent to the disturbance feedback system shown in Fig.6. The disturbance is compensated by adding the estimated disturbance to the current reference as described by (6).

$$\dot{\mathbf{q}}_q(s) = \dot{\mathbf{q}}_q^{ref}(s) + \mathbf{K}_{tn}^{-1} \hat{\boldsymbol{\tau}}_{dis}(s) = \dot{\mathbf{q}}_q^{ref}(s) + \mathbf{K}_{tn}^{-1} \mathbf{G}(s) \boldsymbol{\tau}_{dis}(s) \quad (6)$$

Substituting (6) into (3), the following equation and Fig.7 are obtained.

$$K_{tn}i_q^{ref}(s) = J_n S^2 q(s) + (I - G(s))\tau_{dis}(s) \tag{7}$$

With the feedback of the estimated disturbance, the disturbance vector  $\tau_{dis}(s)$  influences the linear system through the diagonal matrix  $(I - G(s))$ . Therefore, the function  $(I - G(s))$  represents the sensitivity to the disturbance, and is called sensitivity function. The influence of the disturbance becomes small if  $G(s)$  covers the frequency range of the disturbance.

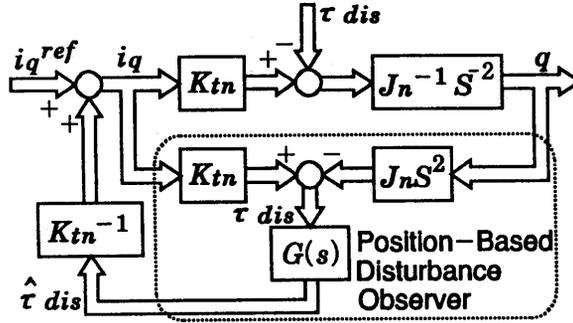


Fig.5: Disturbance compensation by disturbance observer in joint space.

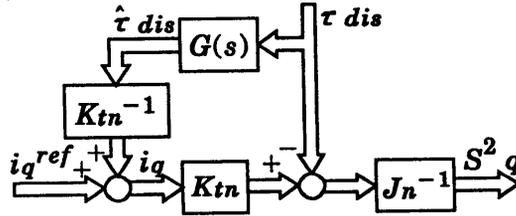


Fig.6: Disturbance compensation in joint space by feedback of disturbance.

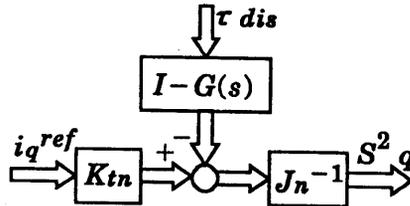


Fig.7: Equivalent block diagram of Fig.3.

The selection of the complementary sensitivity function  $G(s)$  is important because it determines the performance of the control system. The design procedure for  $G(s)$  is shown below.

1. The complementary sensitivity function  $G(s)$  is selected so that the disturbance observers become proper.
2. The sensitivity function  $(I - G(s))$  is determined so as to fulfill the control specifications.

We employ a first-order low-pass-filter  $g/(s + g)$  as  $G(s)$  in order to estimate the disturbance torque from a speed sensor. The comparison of the computation amount of the disturbance observer with that of the inverse kinematics is shown in Table 1. Bode diagram shown in Fig.8 shows that  $|G(s)|$  is nearly unity and  $|1 - G(s)|$  is nearly null at lower frequency. On the other hand, the former is nearly null and the latter is nearly unity at higher frequency. Consequently, the linearized term is dominant at lower frequency; while the disturbance term is not negligible at higher frequency. We can find that the robust system is acquired by low-pass-filter with high cut-off frequency. Such a low-pass-filter, however, cannot suppress the noise of speed sensors or position sensors which exists in high frequency range. Since the noise determines the upper boundary of the cut-off frequency of the low-pass-filter, we have to specify the performance of the sensors so as to fulfill the specifications of the controller.

Table 1: Comparison of computation number for robot dynamics (n:degree-of-freedom).

Method	Newton-Euler	Disturbance Observer
Multiplications	132n	3n
Additions	111n-4	6n

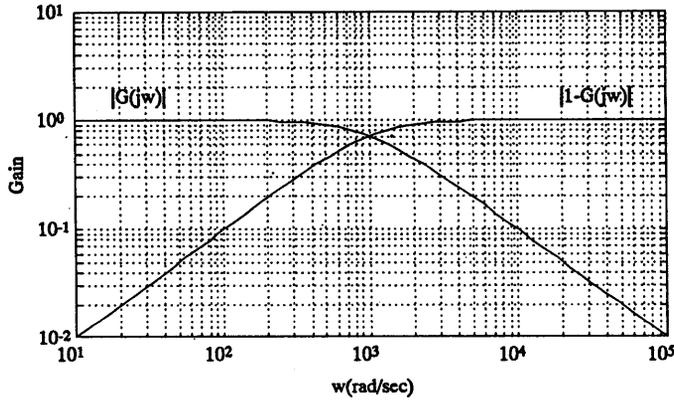


Fig.8: Bode diagram of  $G(s)$  ( $g = 1.0 \times 10^3$ ).

### 2.3 Acceleration Controller in Joint Space

A current reference is calculated by

$$\mathbf{i}_q^{ref}(s) = \mathbf{K}_{tn}^{-1} \mathbf{J}_n \mathbf{S}^2 \mathbf{q}^{ref}(s) \quad (8)$$

to obtain the acceleration controller as shown in Fig.9, because the system is fixed to  $\mathbf{J}_n^{-1} \mathbf{K}_{tn}$  when the influence of disturbance vector  $\boldsymbol{\tau}_{dis}(s)$  is compensated. The acceleration error is given by

$$\mathbf{S}^2 \mathbf{q}^{ref}(s) - \mathbf{S}^2 \mathbf{q}(s) = \mathbf{J}_n^{-1} (\mathbf{I} - \mathbf{G}(s)) \boldsymbol{\tau}_{dis}(s) =: \mathbf{p}(s). \quad (9)$$

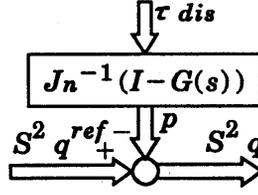


Fig.9: Acceleration controller in joint space.

### 3 Position Control System

#### 3.1 Position Control System in Joint Space

Fig.10 shows a position control system in joint space using the acceleration controller in joint space. The left hand side shows a task space, and the right hand side shows a joint space. The commands in task space are transformed to the joint space by inverse kinematics.

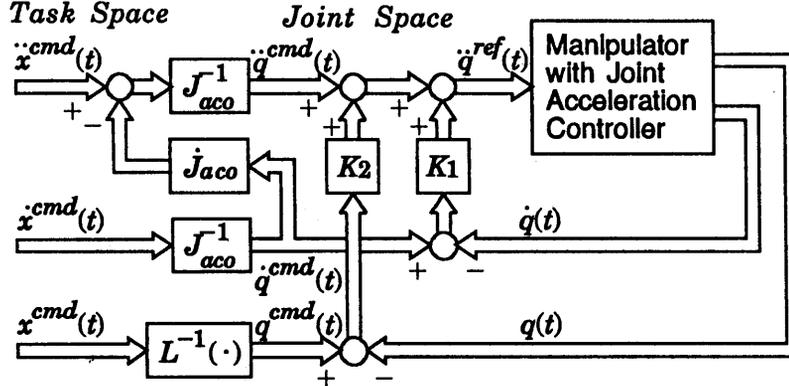


Fig.10: Position control system in joint space using acceleration controller in joint space.

The direct kinematics which relates the generalized coordinates  $q(t)$  to the position vector in task space  $x(t)$  is given by

$$x(t) = L(q(t)). \quad (10)$$

From this equation, the direct kinematics of velocity and acceleration are given by

$$\dot{x}(t) = J_{aco}(q(t))\dot{q}(t) \quad (11)$$

$$\ddot{x}(t) = J_{aco}(q(t))\ddot{q}(t) + \dot{J}_{aco}(q(t))\dot{q}(t) \quad (12)$$

where  $J_{aco}(q(t))$  is Jacobian matrix of  $L(q(t))$ . Inverse kinematics for the position, speed, and acceleration commands are derived from direct kinematics as follows:

$$q^{cmd}(t) = L^{-1}(x^{cmd}(t)) \quad (13)$$

$$\dot{q}^{cmd}(t) = J_{aco}^{-1}(q(t))\dot{x}^{cmd}(t) \quad (14)$$

$$\ddot{q}^{cmd}(t) = J_{aco}^{-1}(q(t))(\ddot{x}^{cmd}(t) - \dot{J}_{aco}(q(t))\dot{q}^{cmd}(t)) \quad (15)$$

The disturbance observer compensates the disturbance of robot manipulators, and realizes the acceleration control. Here, the position controller generates the acceleration reference from position, speed, and acceleration commands.

$$\ddot{q}^{ref}(t) = \ddot{q}^{cmd}(t) + K_1(\dot{q}^{cmd}(t) - \dot{q}(t)) + K_2(q^{cmd}(t) - q(t)) \quad (16)$$

By substituting (16) into (9), the following equation is obtained.

$$\mathbf{q}^{cmd}(s) - \mathbf{q}(s) = (\mathbf{S}^2 + \mathbf{K}_1\mathbf{S} + \mathbf{K}_2)^{-1}\mathbf{J}_n^{-1}(\mathbf{I} - \mathbf{G}(s))\boldsymbol{\tau}_{dis}(s) \quad (17)$$

We find that the position deviation is determined by the interaction among the position controller, sensitivity function by the disturbance observer, and the joint disturbance vector. If the disturbance in each joint is compensated by the disturbance observer as shown in Fig.5, the element of the diagonal matrix of the sensitivity function  $(\mathbf{I} - \mathbf{G}(s))$  becomes high pass filter. Therefore,

$$\lim_{s \rightarrow 0}(\mathbf{I} - \mathbf{G}(s)) = 0. \quad (18)$$

For step disturbance, the final value of the acceleration disturbance shown in (9) becomes null, i.e. the final value of position deviation becomes null.

$$\lim_{t \rightarrow \infty}(\mathbf{q}^{cmd}(t) - \mathbf{q}(t)) = 0 \quad (19)$$

### 3.2 Position Control System in Task Space

Fig.11 shows a position control system in task space using the acceleration controller in joint space. The angular velocity vector and angular position vector are transformed to speed vector and position vector in task space by direct kinematics shown in (10) and (11), respectively. The controller in task space generates the acceleration reference in task space as follows:

$$\ddot{\mathbf{x}}^{ref}(t) = \ddot{\mathbf{x}}^{cmd}(t) + \mathbf{K}_1(\dot{\mathbf{x}}^{cmd}(t) - \dot{\mathbf{x}}(t)) + \mathbf{K}_2(\mathbf{x}^{cmd}(t) - \mathbf{x}(t)) \quad (20)$$

This acceleration reference is transformed to joint space by inverse kinematics as follows:

$$\ddot{\mathbf{q}}^{ref}(t) = \mathbf{J}_{aco}^{-1}(\mathbf{q}(t))(\ddot{\mathbf{x}}^{ref}(t) - \dot{\mathbf{J}}_{aco}(\mathbf{q}(t))\dot{\mathbf{q}}(t)) \quad (21)$$

Since the disturbance observer compensates the disturbance of robot manipulators, and realizes the acceleration control, (21) and (12) are substituted into (9) to obtain the following equation:

$$\ddot{\mathbf{x}}^{ref}(t) - \ddot{\mathbf{x}}(t) = \mathbf{J}_{aco}(\mathbf{q}(t))\mathbf{p}(t) \quad (22)$$

The acceleration error in task space is given by a product of the Jacobian matrix  $\mathbf{J}_{aco}(\mathbf{q}(t))$  and the acceleration error in joint space  $\mathbf{p}(t)$ .

To investigate the characteristics of the position controller, the position controller shown in (20) is substituted into (22) which describes the characteristics of the acceleration control in Cartesian coordinates. Consequently, the equation of the position controller in Cartesian coordinates is given by

$$(\ddot{\mathbf{x}}^{cmd}(t) - \ddot{\mathbf{x}}(t)) + \mathbf{K}_1(\dot{\mathbf{x}}^{cmd}(t) - \dot{\mathbf{x}}(t)) + \mathbf{K}_2(\mathbf{x}^{cmd}(t) - \mathbf{x}(t)) = \mathbf{J}_{aco}(\mathbf{q}(t))\mathbf{p}(t). \quad (23)$$

The position controller is described as the linear system in Cartesian coordinates except for the Jacobian matrix in the disturbance term. Therefore, if the acceleration disturbance vector  $\mathbf{p}(t)$  is null or negligibly small, the position, velocity, and acceleration responses coincide with the commands in both transient and steady states. Moreover, the control in each direction does not interfere in Cartesian coordinates.

Equation(23) is expressed by (24) in Laplace domain and described as shown in Fig.12.

$$\mathbf{x}^{cmd}(s) - \mathbf{x}(s) = (\mathbf{S}^2 + \mathbf{K}_1\mathbf{S} + \mathbf{K}_2)^{-1}\mathcal{L}[\mathbf{J}_{aco}(\mathbf{q}(t))\mathbf{p}(t)] \quad (24)$$

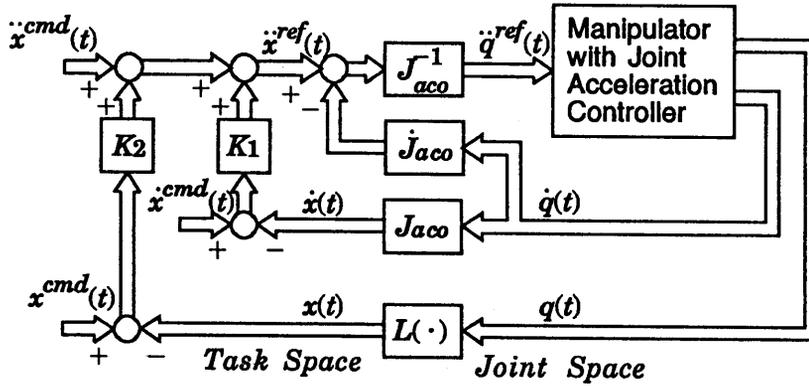


Fig.11: Position control system in task space using acceleration controller in joint space.

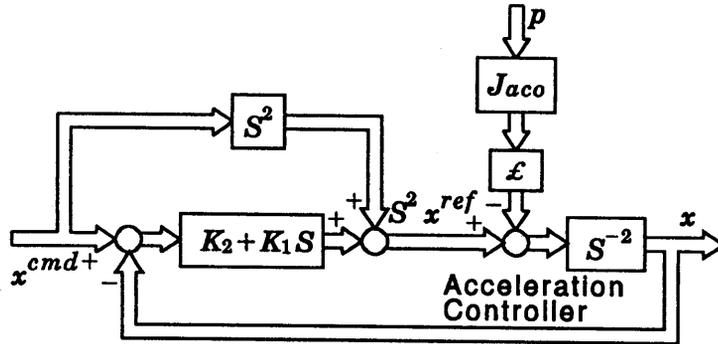


Fig.12: Position controller in Cartesian coordinates.

Considering (24), it is concluded that we find that the position deviation is determined by the interaction between the poles of the position controller and the disturbance term due to the acceleration error in task space  $J_{aco}(q(t))p(t)$ . If the disturbance in each joint is compensated by the disturbance observer as shown in Fig.5, the element of the diagonal matrix of the sensitivity function  $(I-G(s))$  becomes high pass filter. Therefore, for the step disturbance, the final value of the acceleration disturbance shown in (9) becomes null, i.e.

$$\lim_{t \rightarrow \infty} p(t) = 0 \quad (25)$$

From (24), the final value of position deviation becomes null, i.e.

$$\lim_{t \rightarrow \infty} (x^{cmd}(t) - x(t)) = 0 \quad (26)$$

except at the singular point.

## 4 Experimental Results

### 4.1 Experimental System

We show the effectiveness of the proposed position control system by a robot manipulator shown in Fig.13. The total experimental system is shown in Fig.14. Each of 16bit 8086CPU compensates the disturbance of each joint by using the disturbance observer independently in order to realize the acceleration controller for each joint. The DSP (digital signal processor) computes the position controller, the transformations, and acceleration reference signal to each 16bit CPU.

#### 4.2 Experimental Result by Position Control in Joint Space

Fig.15 shows an experimental result of the position control using parameters shown in Table 2. In the case of (a) without the disturbance compensation, the deviation arises due to gravity force, friction force, and so on. In the case of (b), the deviation becomes fairly small, because the disturbance is compensated by the disturbance observer.

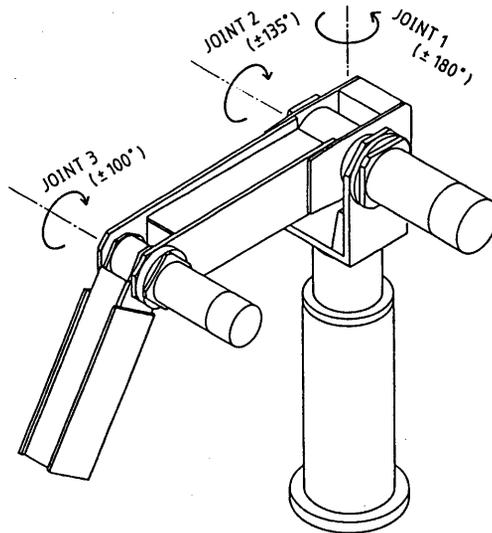


Fig.13: Robot manipulator.

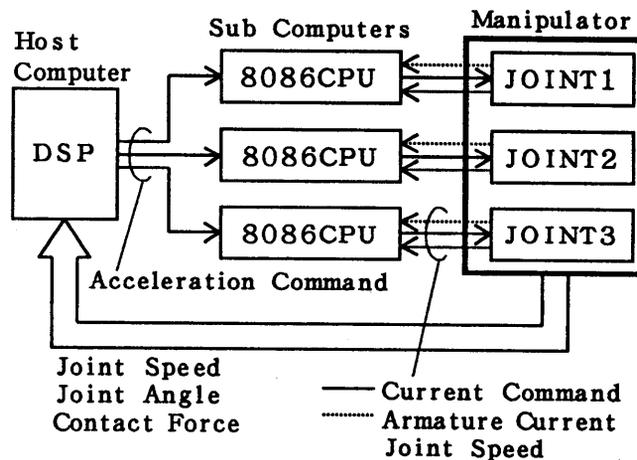
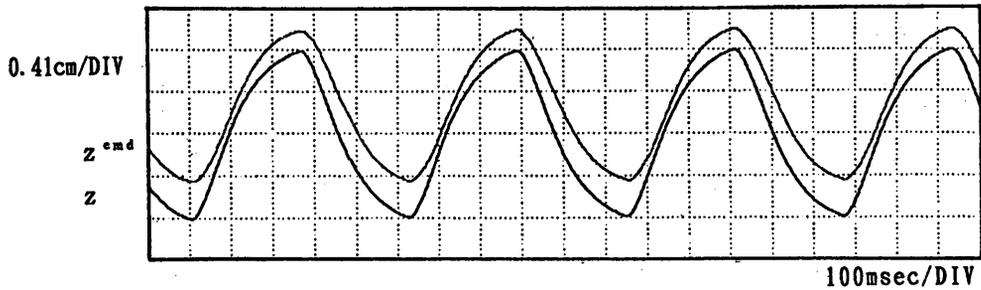


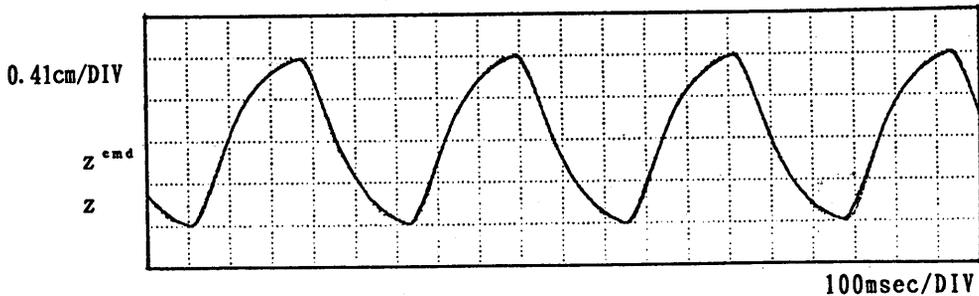
Fig.14: Total system of robot control.

#### 4.3 Experimental Result by Position Control in Task Space

Fig.16 is the experimental result of continuous path control by the position controller shown in Fig.11. The result shows that the position response  $y$  coincides with the command  $y^{cmd}$ ; and the maximum velocity deviation  $\dot{y}^{cmd} - \dot{y}$  is about 2cm/s. We find that each position controller is robust against the disturbance, and has good response with little interference from the other direction in Cartesian coordinates.



(a) without disturbance observer.



(b) with disturbance observer.

Fig.15: Experimental result of position control in joint space.

Table 2: Control parameters.

$diag(K_1)[s^{-1}]$	90
$diag(K_2)[s^{-2}]$	2,000
$diag(g)[rad/s]$	700
control period[msec]	1

## 5 Conclusion

This paper presented the robust position control system of robot manipulators. In joint space, the disturbance observer linearizes robot manipulators. Since the acceleration can be controlled by using the disturbance observer, robust and fast position control is realized. The usefulness of the proposed control system is confirmed by the experiments.

Two types of position controllers were presented in this paper. When the trajectory of the path is known, the control period can be reduced by using the position controller in joint space. The trajectory in joint space is determined by off-line computing the inverse kinematics of the trajectory.

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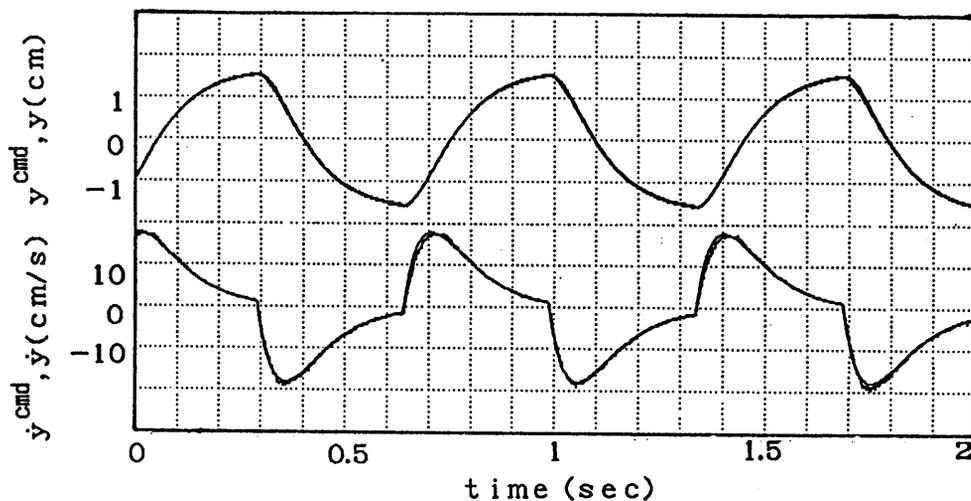


Fig.16: Experimental result of position control in task space.

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