Res. Rep. Fac. Eng. Mie Univ., Vol. 27, pp. 37－46 (2002)

Original Paper

Elastic Buckling Strength of Braced Parallel Compression Members

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(Received September 27, 2002)

Abstract

This paper deals with the elastic buckling strength of a system consisting of a number of centrally-loaded compression members with the same length, which are connected each other by rigid members and elastically braced at midpoint. The compression load and bending stiffness of each member are set equal to s_i and t_i times those of the Reference member, respectively. An approximate method to compute elastic buckling strength of the system is proposed, by replacing the system by an equivalent single braced member, and the accuracy of the approximate solution are discussed, based on the results of analysis of several examples.

Keywords: parallel compression members, braced member, buckling strength, approximate solution

1. Introduction

Steel structural member is often strengthened by arranging intermediate supporting points, which makes the effective buckling length of the member shorter. Behavior of a braced compression member have been extensively investigated, and strength of the member and strength and stiffness required for the brace have been clarified to a certain extent. Theoretical works in Refs. [1] through [13] and experimental works in Refs. [14] through [18] have investigated the effects of slenderness of the compression member, stiffness, position and number of the braces, and initial imperfection on strength of the compression member and reaction force generated in the brace. These works have all dealt with a single compression member, but it is encountered in a real practice that parallelly-arranged several members are connected each other and braced all together; for example, roof beams braced by purlins, and outer columns braced by furring strips in a warehouse or factory building. The bracing requirements for such cases must be investigated by treating those members as a total system, but very few investigation has been done so far[19, 20].

The purpose of research presented in this paper is to develop an evaluation method of the elastic buckling strength of a system of parallelly-braced compression members. The paper first introduces an exact method of buckling analysis of the system, and proposes an approximate method of analysis by replacing the system by an equivalent single braced member. Finally, characteristics of the buckling strength of the parallelly-braced compression members and accuracy of the approximate solution are discussed with several nuerical examples.

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2. Mathematical Model and Method of Analysis

2.1 Formulas of Slope-Deflection Method Considering Axial Force Effect

Figure 1 shows a compression member ij, being pin-supported at end i and rigidly-connected to adjacent member at end j, which buckles under the axial force N. Member-end moment M and lateral force V occurs, and the member deflects with member-end slope θ and chord rotation angle R. Subscripts i and j indicates the quantities occurring at ends i and j, respectively. Positive directions of forces and deformations are taken as shown in Fig. 1. Formulas of slope-deflection method considering axial force effect for member-end moments and lateral forces of the member ij in Fig. 1 are written as follows:

$$M_{ij} = \frac{EI}{l}(\xi\,\theta_i - \xi\,R) \tag{1}$$

$$V_{ij} = V_{ji} = -\frac{EI}{l^2}(\xi\,\theta_i - \omega\,R) \tag{2}$$

where E, I and l denote Young's modulus, moment of inertia and length of the member, respectively, and ξ and ω are stability functions of axial force parameter Z, given as follows:

$$\xi = \frac{Z^2 sinZ}{sinZ - Z\,cosZ}\ , \qquad \omega = \frac{Z^3 cosZ}{sinZ - Z\,cosZ} \tag{3}$$

$$Z = l\sqrt{\frac{P}{EI}} \tag{4}$$

2.2 Mathematical Model for Analysis

Figure 2 shows a mathematical model which represents a system of parallelly-braced compression members. n members with the same length $2l$ are parallelly arranged, each one being pin-supported at both ends. Member i has flexural stiffness t_iEI, and is subjected to the axial load s_iP, where $t_1 = s_1 = 1$, and member 1 is chosen so that the condition $s_i/t_i \leq 1$ is satisfied. Each member is connected together at midheight by rigid bars without restraining the rotation, and lateral displacement of at midheight of the total system is restrained by an elastic spring with spring constant K.

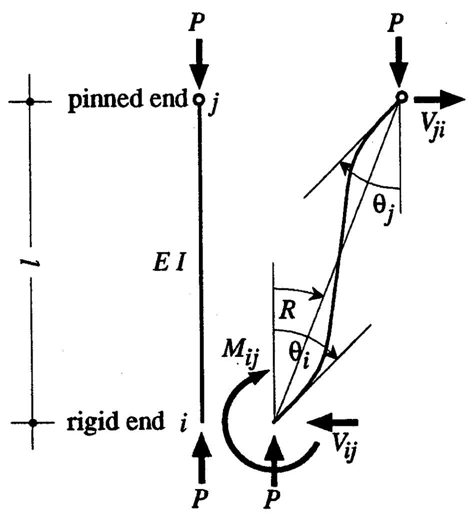

Fig. 1 Deformed Member Subjected to End Forces

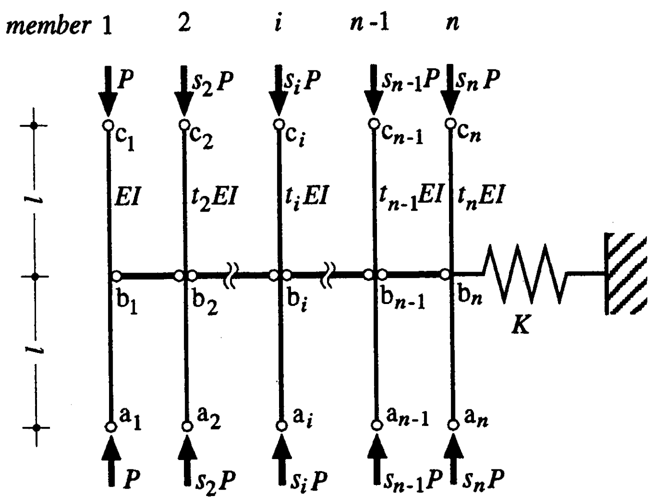

Fig. 2 Model for Analysis

2.3 Exact Analysis of Buckling Strength

Figure 3 shows member i separated from the system, in which slope θ_i at midheight and chord rotation R in the portion a_ib_i occur. Applying the slope-deflection formula, Eq. (2), for the portions a_ib_i and b_ic_i leads to the expressions of the member-end lateral forces V_{biai} and V_{bici} as follows:

$$V_{b_i a_i} = -\frac{t_i EI}{l^2}(\xi_i\theta_i - \omega_i R), \qquad V_{b_i c_i} = -\frac{t_i EI}{l^2}(\xi_i\theta_i + \omega_i R) \quad (i = 1, 2, \cdots, n) \tag{5}$$

where

$$\xi_i = \frac{Z_i^2 sinZ_i}{sinZ_i - Z_i cosZ_i}, \qquad \omega_i = \frac{Z_i^3 cosZ_i}{sinZ_i - Z_i cosZ_i} \tag{6}$$

$$Z_1 = l\sqrt{\frac{P}{EI}}, \quad Z_i = l\sqrt{\frac{s_i P}{t_i EI}} = \sqrt{\frac{s_i}{t_i}}\, Z_1 \quad (i = 2, 3, \cdots, n) \tag{7}$$

Difference between V_{biai} and V_{bici} is given by

$$V_{b_i a_i} - V_{b_i c_i} = \frac{2EI}{l^2} t_i \omega_i R \quad (i = 1, 2, \cdots, n) \tag{8}$$

is transmitted to and supported by the spring, as shown in Fig. 4. Thus, the equilibrium of the lateral forces for the total system is given by

$$F + (V_{b_1 a_1} - V_{b_1 c_1}) + (V_{b_2 a_2} - V_{b_2 c_2}) + \cdots + (V_{b_n a_n} - V_{b_n c_n}) = 0 \tag{9}$$

where F is the supporting force at the spring and given by

$$F = K\Delta = KlR \tag{10}$$

Substituting Eqs. (8) and (10) into (9) leads to

$$KlR + \frac{2EI}{l^2}(\omega_1 + t_2\omega_2 + \cdots + t_n\omega_n)R = 0 \tag{11}$$

Taking R out from Eq. (11),

$$\frac{Kl^3}{2EI} + \omega_1 + t_2\omega_2 + \cdots + t_n\omega_n = 0 \tag{12}$$

Introducing non-dimensional spring constant k shown below,

$$k = \frac{K}{K_0} = \frac{Kl^3}{2\pi EI}; \quad K_0 = \frac{2P_E}{l}; \quad P_E = \frac{\pi^2 EI}{l^2} \tag{13}$$

the buckling condition of the system shown in Fig. 2 is finally obtained as follows:

$$\pi^2 k + \sum_{i=1}^{n} t_i\omega_i = 0 \tag{14}$$

Note that ω_i and Z_i are given by Eqs (6) and (7), and Z_1 is numerically the largest among the load parameter Z_i's, since $t_1 = s_1 = 1$, and $s_i/t_i \le 1$. The smallest value of Z_1 that satisfies Eq. (14) gives the buckling strength P_{cr}.

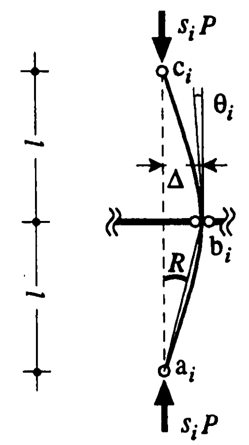

Fig. 3 Deformation of Member i

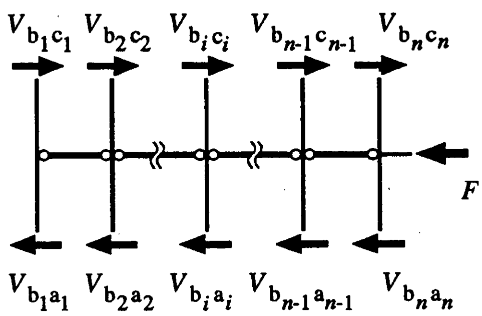

Fig. 4 Equilibrium of Lateral Forces

2.4 Approximate Analysis of Buckling Strength

In the system shown in Fig. 2, the value of Z_1 is the largest, and this means that members 2 through n whose values of Z_i's are all less than Z_1 tend to support the most critical member 1 at the moment of buckling. In other words, if $t_i = s_i$ in all members, all the values of Z_i's are identical, and the buckling of all members occur simultaneously. However, since $s_i/t_i \leq 1$, remaining stiffness, $(t_i - s_i)\,EI$, may be utilized to brace the total system. Along this context, an approximate method of analysis is considered in this section.

Figure 5(a) shows a fictitious compression member braced at the midheight by a spring with the spring constant K'_i, which simulates the condition of member i of the model shown in Fig. 2, and K'_i is a part of K. If the stiffness of member i is s_iEI instead of t_iEI, all the values of Z_i's become identical and equal to Z_1, as shown below.

$$Z_i = l\sqrt{\frac{s_i P}{s_i E I}} = l\sqrt{\frac{P}{EI}} = Z_1 \quad (i = 2, 3, \cdots, n) \tag{15}$$

Figure 5(b) shows a replacement of the member in Fig. 5(a) to a loaded member and a member without load, which are connected together by a rigid bar and braced by the spring K'_i. Treating the remaining stiffness $(t_i - s_i)EI$ as an additional spring leads to another replacement shown in Fig. 5(c). The spring constant $\overline{K}_i$ of the additional spring is obtained from the stiffness of a simple beam subjected to a lateral load at the center as shown in Fig. 6, as follows:

$$\overline{K}_i = \frac{6\,(t_i - s_i)E\,I}{l^3} \qquad (i = 2, 3, \cdots, n) \tag{16}$$

Suppose each of n members in Fig. 2 are replaced to the one shown in Fig. 5(c), total stiffness of all springs becomes

$$K'_1 + K'_2 + \cdots + K'_n + \overline{K}_2 + \cdots + \overline{K}_n = \sum_{i=1}^{n} K'_i + \sum_{i=2}^{n} \overline{K}_i = K + \sum_{i=2}^{n} \overline{K}_i \tag{17}$$

Then, the model shown in Fig. 2 may be replaced to the system shown in Fig. 7. Since the value of the load parameter Z is identical in all the members, when the system shown in Fig. 7 buckles, this system is considered as an assembly of n single members braced at the center, whose members 1 and i are shown in Fig. 8, where the spring constants K_1 and K_i are part of the original constant K, and they have the values necessary to make the buckling strength of both single members equal to Z. In other words, the system shown in Fig. 7 may be broken down into n single members shown in Fig. 8, both of which buckle at $Z_1 = Z_i = Z$, and K_i must satisfy following condition.

$$K_1 + K_2 + \cdots + K_n = K + \sum_{i=2}^{n} \overline{K}_i \tag{18}$$

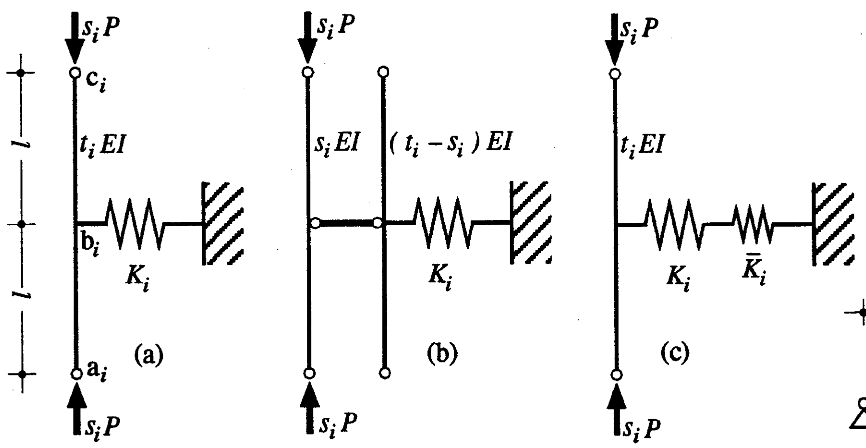

Fig. 5 Replacement of Member i

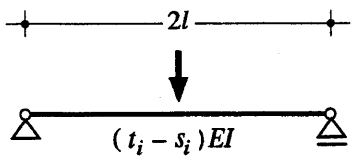

Fig. 6 Evaluation of Additional Spring

The buckling conditions for two members shown in Fig. 8 are given as follows, Referring to Eq. (12):

$$\frac{K_1 l^3}{2EI} + \omega = 0, \qquad \frac{K_i l^3}{2EI} + s_i\,\omega = 0 \quad (i = 2, 3, \cdots, n) \tag{19}$$

where ω is given by Eq. (3), noting that $Z_1 = Z_i = Z$. Equation (19) is re-written as

$$\frac{K_1 l^3}{2\pi^2 EI} = -\frac{\omega}{\pi^2}, \qquad \frac{K_i l^3}{2\pi^2 EI} = -s_i \frac{\omega}{\pi^2} \quad (i = 2, 3, \cdots, n) \tag{20}$$

As mentioned above, Eq. (20) may be recognized as the requirements for the spring constants to make each of single members braced at he center as shown in Fig. 8, which are separated from the system shown in Fig. 7, buckle simultaneously at the load parameter equal to Z. From Eq. (20), relation between K_1 and K_i is given as

$$K_i = s_i K_1 \quad (i = 2, 3, \cdots, n) \tag{21}$$

Substituting Eqs. (16) and (21) into Eq. (18) leads to

$$(1 + s_2 + \cdots + s_n)K_1 = K + \frac{6EI}{l^3}\{(t_2 - s_2) + (t_3 - s_3) + \cdots + (t_n - s_n)\} \tag{22}$$

Non-dimensionalizing Eq. (22) by Eq. (13) leads to

$$k_1 = \frac{k + \dfrac{3}{\pi^2}\sum\limits_{i=1}^{n}(t_i - s_i)}{\sum\limits_{i=1}^{n} s_i} \tag{23}$$

Finally, the buckling strength of a system of parallelly-braced compression members shown in Fig. 2 is approximated by the strength of a single compression member shown in the left of Fig. 8, whose spring constant K_1 is given by Eq. (23), and the buckling condition is given as follows:

$$K = (1 + s_2 + \cdots + s_n)K_1 - (\overline{K_2} + \overline{K_3} + \cdots + \overline{K_n}) \tag{24}$$

where w is given by Eq. (3).

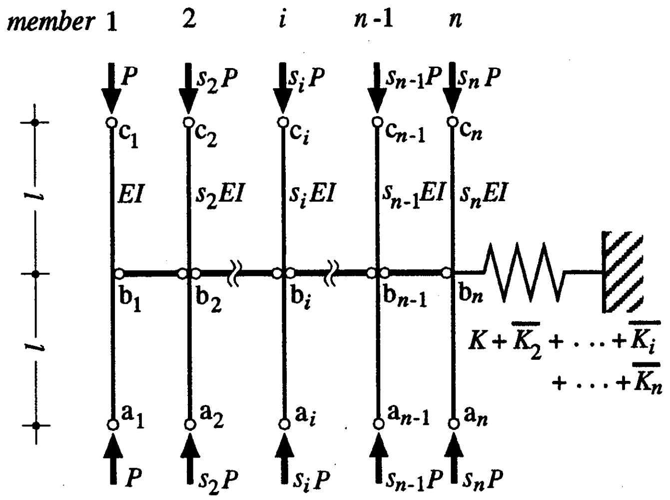

Fig. 7 Replaced Model with Additional Springs

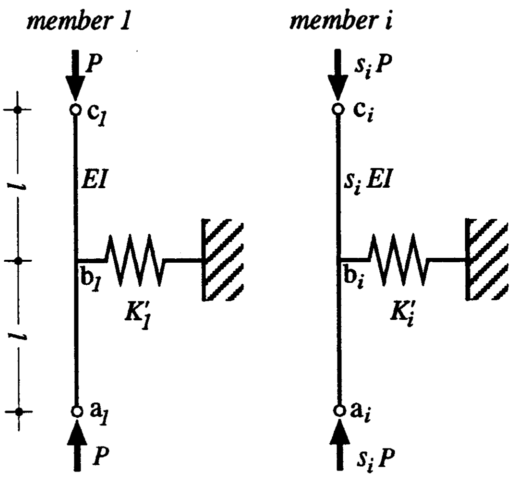

Fig. 8 Isolated Members 1 and i

3. Results of Analysis and Discussion

Figures 9, 10 and 11 shows examples of parallelly-braced compression members numerically analyzed, which deal with the case of $n = 2$, $n = 3$, and $n = 3$, 5 and 10, respectively. Intensity of axial load is identical in each member in the case of examples with odd number, while the value of EI is identical in the case of examples with even number.

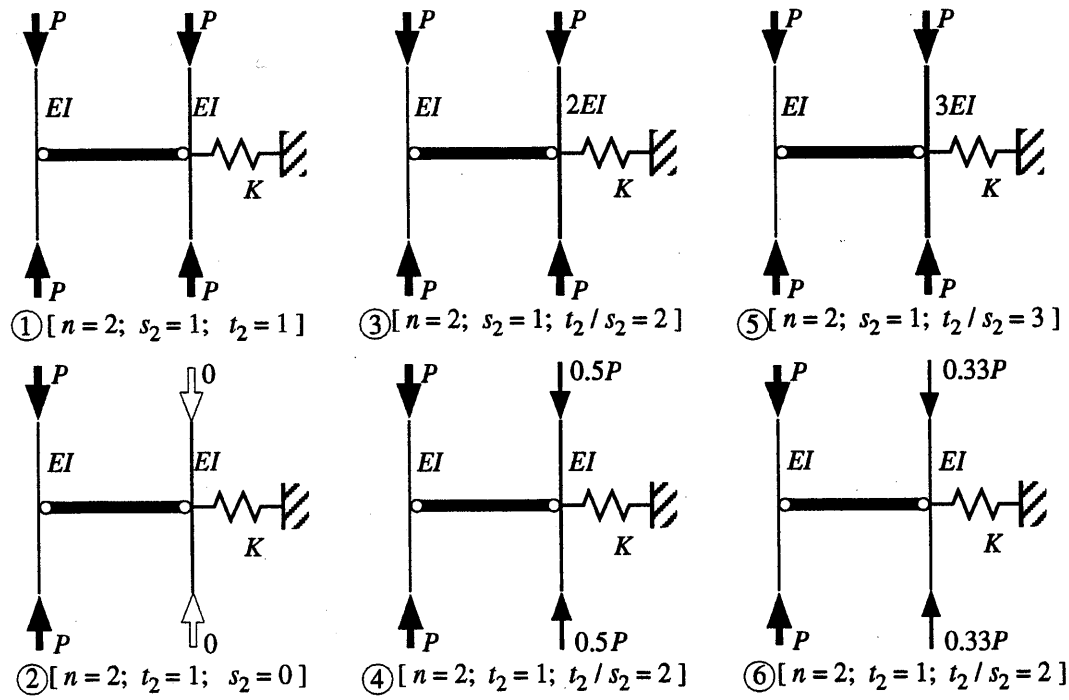

Fig. 9 Numerical Examples ($n = 2$)

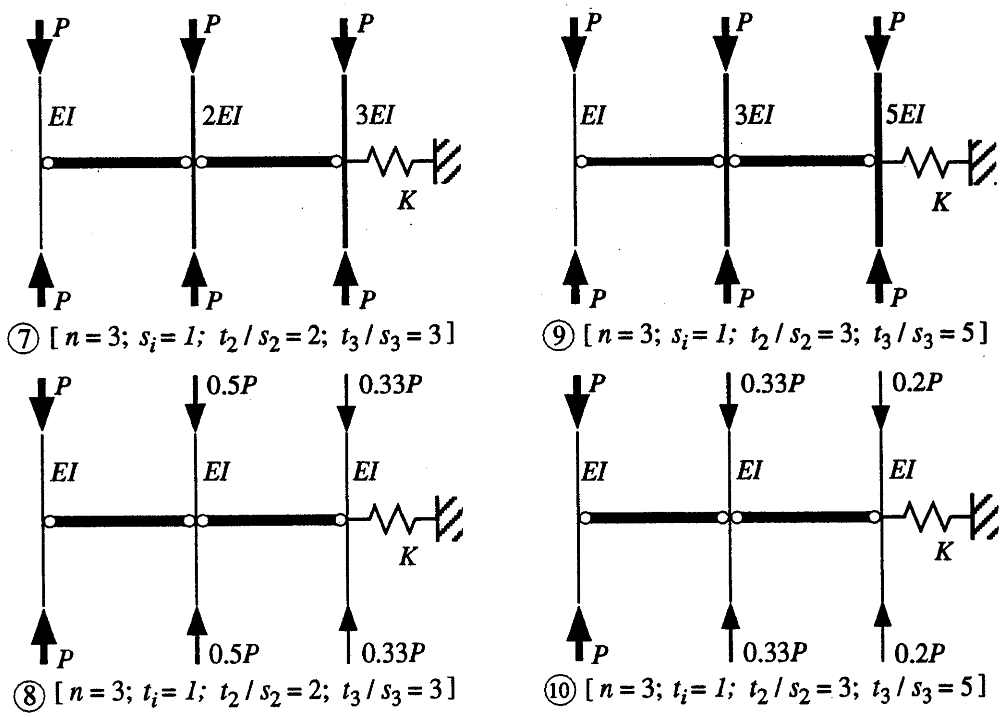

Fig. 10 Numerical Examples ($n = 3$)

Results of numerical computation of the buckling strength of examples are shown in Figs. 12 through 17, where the values of the buckling load parameter Z_{cr} are plotted against the values of the non-dimensional spring constant k. Figure 12 shows the effects of load ratio si and stiffness ratio ti on Z_{cr}-k curves for n = 2. Figures 13 and 14 show the effects of the ratio t_i/s_i on Z_{cr}-k curves for n = 2 and n = 3, respectively. These Figures also show the results of a single member braced at the center for comparison, for which the value of k_1 is taken for abscissa. Figures 15 and 16 compares Z_{cr}-k curves obtained by exact (thick lines) and approximate (thin lines) analyses for n = 2 and n = 3, respectively. Figure 17 shows the change of Z_{cr}-k curves with the change in the value of n, comparing exact and approximate strengths. Numbers of examples appearing in these Figures correspond to those in Figs. 9 through 11.

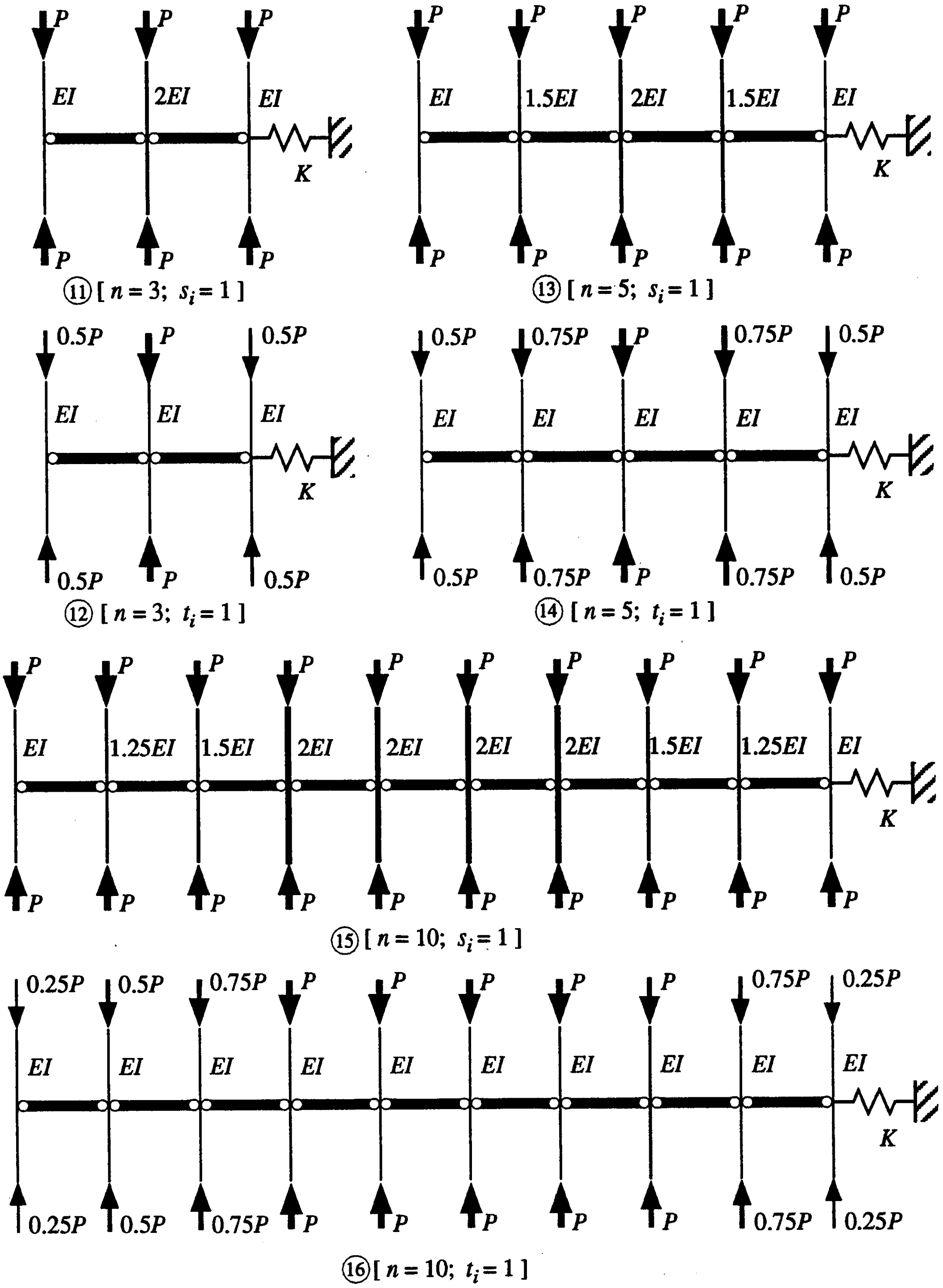

Fig. 11 Numerical Examples (n = 3, 5, 10)

In general, the value of Z_{cr} increases with the increase in k, but it becomes equal to π for k larger than a certain value, and the buckling mode becomes one-sine wave at this stage, without causing reaction force in the spring. If the value of k is the same, the system with smaller s_i and larger t_i shows larger value of Z_{cr}. It is observed from Figs. 13 and 14 that there exists a certain value of k at which the value of Z_{cr} of two systems with identical value of t_i/s_i becomes nearly equal, and this is also nearly equal to the value of Z_{cr} of a single member. It is observed

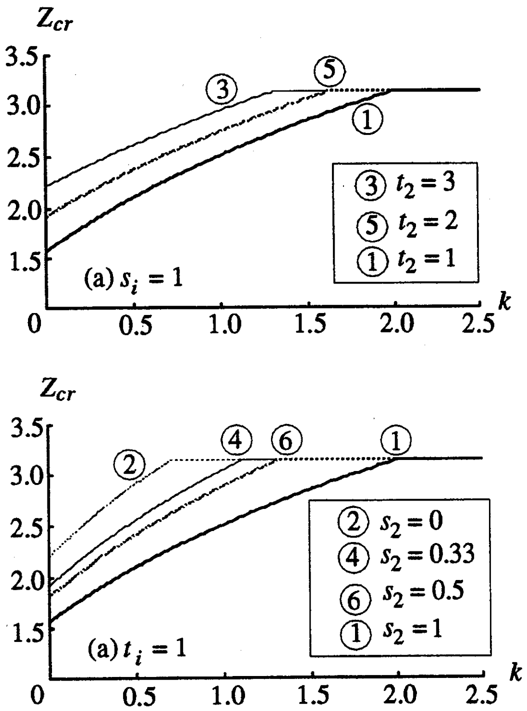

Fig. 12 Effects of Load Ratio and Stiffness Ratio ($n = 2$)

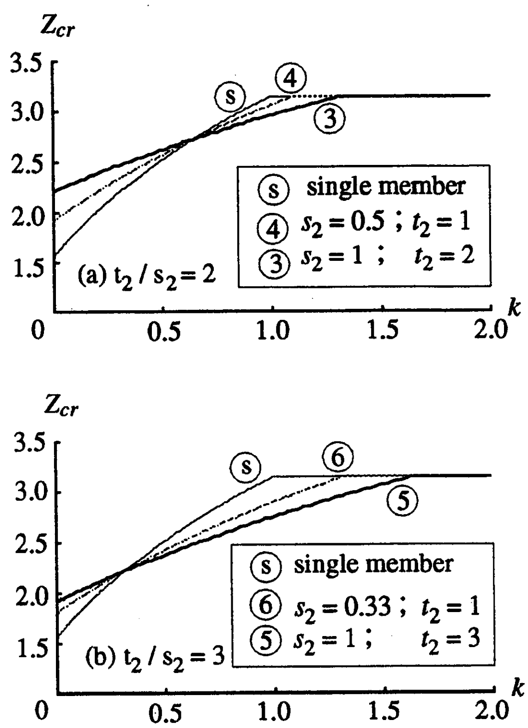

Fig. 13 Comparison of Z_{cr}- k Curves t_i/s_i = const. ($n = 2$)

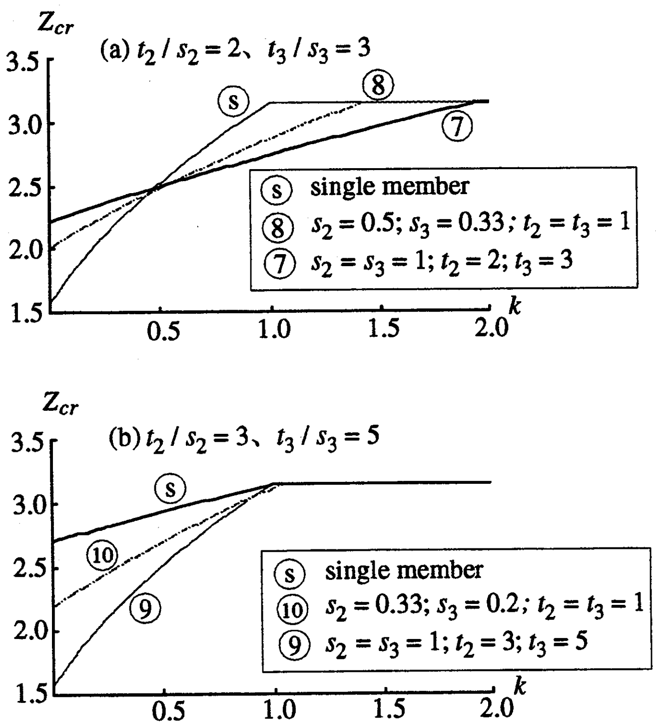

Fig. 14 Comparison of Z_{cr}- k Curves: t_i/s_i = const. ($n = 3$)

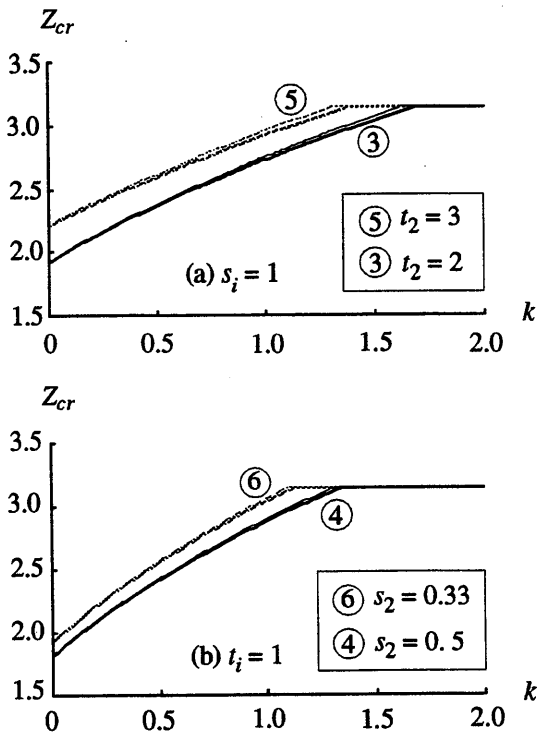

Fig. 15 Comparison of Exact and Approximate Solutions ($n = 2$)

from Figs. 15 and 16 that the approximate solution shows fairly good accuracy, and gives conservative estimate to the exact solution. Accuracy seems to become worse with the increase in k, and worse in the case of $s_i = 1$ than in the case of $t_i = 1$. Accuracy is still very good when the number of compression members n increases to 10 as shown in Fig. 17, where the maximum error is about 1.2% in the case of $s_i = 1$, and 0.8% in the case of $t_i = 1$.

4. Conclusions

i) Exact buckling condition for a system of parallelly-braced compression members treated in this study is given in a very simple form, Eq. (14), regardless of the number of compression members.

ii) An approximate method has been proposed to evaluate the buckling strength of a system of parallelly-braced compression members, which brakes down the given system to an equivalent single compression member braced at the center by a spring, whose spring constant is given by a function of the spring constant, load ratio and stiffness ratio of the given system.

iii) Accuracy has been tested through numerical computation of several examples of the system of parallelly-braced compression members, and verified fairly good; the maximum error comparing with the exact solution of an example consisting of 10 compression members is about 1.2% in the case of identical load ratio, and 0.8% in the case of identical stiffness ratio.

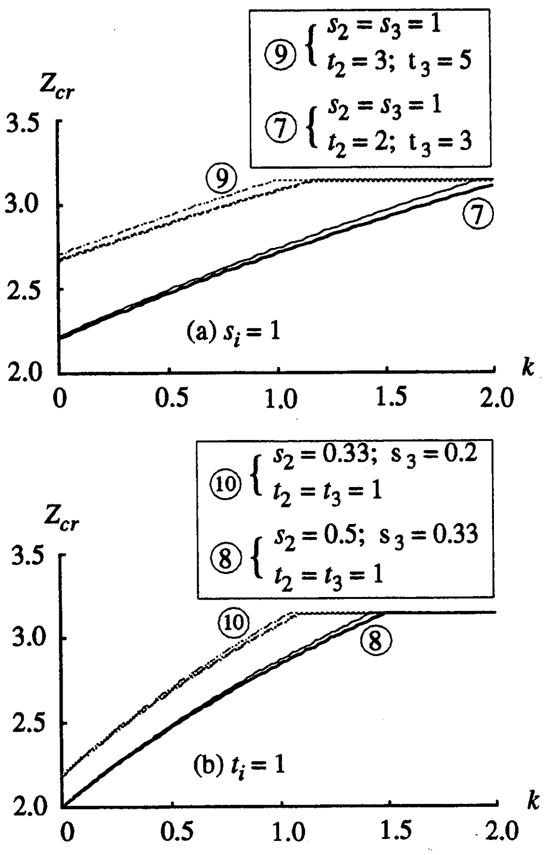

Fig. 16 Comparison of Exact and Approximate Solutions ($n = 3$)

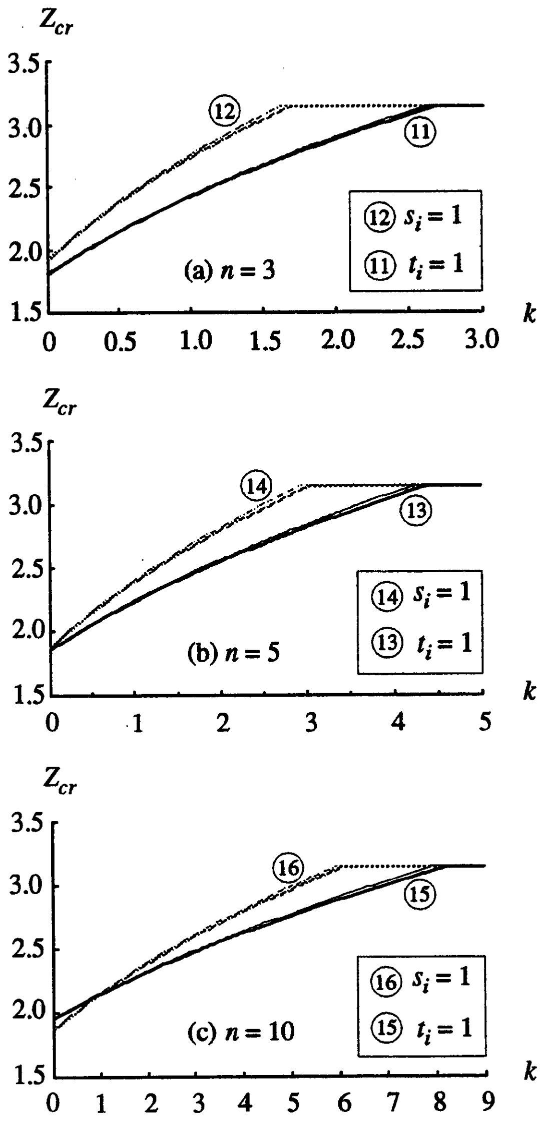

Fig. 17 Comparison of Exact and Approximate Solutions ($n = 2, 3, 5$)

References

[1] Zuk, W.: Lateral Bracing Forces on Beams and Columns, Journal of the Engineering Mechanics Division, Proceedings of the American Society for Civil Engineers (ASCE), Vol. 82, No. EM3, pp. 1032-1 - 1032-11, 1956.3.

[2] Winter, G.: Lateral Bracing of Columns and Beams, Transactions of ASCE, Vol. 125, Part I, pp. 807-845, 1960.

[3] Saisho, M., Tanaka, H., Takanashi, K. and Udagawa, K.: Lateral Bracing of Compression Members, Transactions of the Architectural Institute of Japan (AIJ), No. 184, pp. 73-79, , 1971.6. (in Japanese)

[4] Matsui, C. and Matsumura, H.: Study on Lateral Bracing of Axially Compressed Members, Part 1 An Elastic Plastic Analysis, Trans. of AIJ, No. 205, pp. 23-29, 1973.3. (in Japanese)

[5] Ono, T., Ishida, T. and Shimono, K.: A Study on Bracing for Steel Column and Beam Considering Limit State, Journal of Structural and Construction Engineering, Trans. of AIJ, No. 469, pp. 117-125, 1995.3. (in Japanese)

[6] Nishino, T. and Tsuji, B.: Behavior of Compression Members with Lateral Bracing, J. of Struct. Constr. Eng., Trans. of AIJ, No. 483, pp. 157-163, 1996.5. (in Japanese)

[7] Ono, T., Ishida, T. and Shimono, K.: A Study on Bracing Requirement of Steel Beam-Column, Proc. 5th International Colloquium on Stability and Ductility of Steel Structures, Nagoya, Vol. 1, pp. 507-514, 1997.7.

[8] Tsuji, B. and Nishino, T.: Post-Buckling Strength of Compression Members with Elastic Plastic Lateral Bracing, Proc. 5th International Colloquium on Stability and Ductility of Steel Structures, Nagoya, Vol. 1, pp. 515-522, 1997.7.

[9] Nishino, T. and Tsuji, B.: Post-Buckling Behavior of Compression Members with Elastic Plastic Lateral Bracing, J. of Struct. Constr. Eng., Trans. of AIJ, No. 501, pp. 135-142, 1997.11. (in Japanese)

[11] Nishino, T. and Tsuji, B.: Behavior of Compression Members with Two Lateral Bracings, J. of Struct. Constr. Eng., Trans. of AIJ, No. 502, pp. 119-126, 1997.12. (in Japanese)

[12] Fukao, H., Morino, S. and Kawaguchi, J.: Elasto-Plastic Behavior of Laterally-Braced Compression Members, Stability and Ductility of Steel Structures, Edited by T. Usami and Y. Itoh, Elsevier, pp.37-44, 1998.1.

[13] Fukao, H. and Morino, S.: Elasto-Plastic Behavior of Compression Members Laterally-Braced at an Intermediate point and Bracing Requirements, J. of Struct. Constr. Eng., Trans. of AIJ, No. 528, pp. 151-157, 2000.2. (in Japanese)

[14] Suzutani, J.: Inelastic Buckling of Members with Intermediate Support, Summaries of Technical Papers of Annual Meeting of AIJ, Structures Division, pp. 965-966, 1968.10. (in Japanese)

[15] Suzutani, J. and Kikuchi, F.: Buckling of Compression Members with Intermediate Support, Summaries of Technical Papers of Annual Meeting of AIJ, Structures Division, pp. 1037-1038, 1969.8. (in Japanese)

[16] Matsui, C. and Matsumura, H.: Study on Lateral Bracing of Axially Compressed Members, Part 2 An Experimental Study of Members with Rectangular Cross Section, Trans. of AIJ, No. 208, pp. 15-21, 1973.6. (in Japanese)

[17] Saisho, M.: Tests on Lateral Bracing of Steel Members Subjected to Repeated Load, Summaries of Technical Papers of Annual Meeting of AIJ, Structures Division, pp. 1331-1332, 1978.9. (in Japanese)

[18] Fukao, H., Kuwada, S., Morino, S. and Kawaguchi, J.: Experimental Investigation on Elasto-Plastic Behavior of Compression Members with an Intermediate Elastic Lateral Support, J. of Struct. Constr. Eng., Trans. of AIJ, No. 530, pp. 171-176, 2000.4. (in Japanese)

[19] Shigeta, I. and Nakamura, T.: Fundamental Study of Lateral Bracing for Parallel Compressive Members, Part I, Summaries of Technical Papers of Annual Meeting of AIJ, C-1, Structure III, pp. 181-182, 1995.8. (in Japanese)

[20] Shigeta, I. and Nakamura, T.: Fundamental Study of Lateral Bracing for Parallel Compressive Members, Part II, Summaries of Technical Papers of Annual Meeting of AIJ, C-1, Structure III, pp. 285-286, 1996.9. (in Japanese)