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A Theoretical Study on the Effect Calculation of Artifical Reef

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Abstract

A simple method which uses the catch and effort statistics collected before and after the setting of an artificial reef is proposed for calculating the effect capacity of the reef. Following differential equation on the stock and fishing effort was assumed:

$$\frac{dS}{dt} = kN\,(C - S) - y, \quad \frac{df}{dt} = a - hCf + hy$$

where S = stock quantity existing on an artificial reef, f = fishing mortality coefficient, C = maximum capacity of an artificial reef, N = stock size migrating to the fishing ground, y = quantity of fish caught, $a = hCf_{\max}$, and k and h denote coefficient.

When S and f are in equilibrium, $\widehat{f} = a / hC + \widehat{y} / C$ is led. The exploitation rate f_i and catch quantity y_i for time interval i give curve with continuous oscillation around an equilibrium. Applying an method of moving average, the following approximate expression can be formed:

$$\tilde{\tilde{f}}_i = a / hC + \tilde{\tilde{y}}_i / C.$$

If f_i and y_i during several years before and after the setting can be obtained, linear regression coefficient C (maximum capacity of an artificial reef) will be found.

Key words: artificial reef, effect calculation, maximum capacity

The criteria for calculating the effect of an artificial reef can roughly be grouped into: (1) production effect (catch effect); and (2) fish-attracting effect. The latter is calculated mainly by direct methods, such as visual observation, use of echo sounders and experimental fishing, while the former is generally calculated by indirect methods in which analyses are made of the information about sea areas where the artificial reef has been set, such as catch and effort statistics, and fishing operation records, etc.

Brock[1)] and Samples and Sproul[2)] calculated the effect of an artificial reef by some methods. In the present work, the authors will describe an attempt to calculate the effect by the use of the catch and effort statistics collected before and after the setting of an artificial reef.

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Artificial Reef Effect and Expression of Fishing Mortality Coefficient

It is difficult to define how far an artificial reef could exercise its effect on the stock. Assuming that the principal part of the effect exists in artificial reef itself, however, there will inevitably be a certain limit to the effect capacity of an artificial reef depending on fish species.

Let C be the maximum capacity of an artificial reef, and S the stock quantity currently existing on the reef. Then the quantity of fish newly added to the existing stock can be expressed in the form of the product of the size of the stock migrating to the fishing ground, N and the surplus capacity of the reef, $(C-S)$. If the coefficient of addition made by the individuals which have come across the reef is represented by k, and the fishing mortality coefficient of the stock caught on this ground by f, then the quantity of fishing caught, y can be expressed by Equation $y = f \cdot S$, and the change of the stock quantity, $\frac{dS}{dt}$ by Equation (1),

$$\frac{dS}{dt} = k \cdot N(C-S) - fS = k \cdot N(C-S) - y. \tag{1}$$

The increasing or decreasing trend of the fishing mortality coefficient $\left(\frac{df}{dt}\right)$ may be expressed by the sum of (1) what is always striven for to achieve maximum catch quite independently of the stock or catch quantities and (2) what is dependent of catch quantity ($h \cdot y$; h denotes coefficient). If f_{max} denotes the maximum fishing mortality coefficient that can be assigned to a certain fish species in a certain fishing ground, (1) is expressed by $h \cdot C(f_{max} - f)$. If $h \cdot C \cdot f_{max} = a$, a is a constant which represents the trend of the maximum fishing mortality coefficient and the following formula is led;

$$\frac{df}{dt} = a - h \cdot C \cdot f + hy. \tag{2}$$

Theoretical Development

The values of S and f in an equilibrium state can be obtained by the following Equations if the right side of Eqs. (1) and (2) is assumed 0:

$$\widehat{S} = \frac{1}{k \cdot N}(k \cdot NC - \widehat{y}) \tag{3}$$

$$\widehat{f} = \frac{1}{h \cdot C}(a + h \cdot \widehat{y}) \tag{4}$$

where $\widehat{y}$ indicates catch quantity when S and f are in equilibrium. And from (3) $\times$ (4) is obtained $h\widehat{y}^2 + a\widehat{y} - k \cdot NCa = 0$ and from this, $\widehat{y} = \frac{1}{2h}[-a + \sqrt{a^2 + 4k \cdot h \cdot NCa}]$ is obtained.

On the other hand, if

$$S = \widehat{S} + S^* \tag{5}$$

$$f = \widehat{f} + f^* \tag{6}$$

are assumed with respect to the property of the solutions in the neighborhood of the equilibrium and if these Equations are substituted into Eqs. (1) and (2), where minor terms of an order higher than those of S^* and f^* are omitted, then $\frac{dS^*}{dt} = -S^*(k \cdot N + \widehat{f}) - f^*\widehat{S}$ and $\frac{df^*}{dt} = h \cdot \widehat{f}S^* - h(C - \widehat{S}) \cdot f^*$ are obtained. The solutions of these Equations are usually obtained by assuming $S^* = A_1 e^{\lambda t}$, $f^* = A_2 e^{\lambda t}$, and their general solutions are:

$$S^* = (B_{11} \cdot \cos \tau t + B_{12} \cdot \sin \tau t)\ e^{\sigma t} \tag{7}$$

$$f^* = (B_{21} \cdot \cos \tau t + B_{22} \cdot \sin \tau t)\ e^{\sigma t} \tag{8}$$

where $\tau^2 < 0$. From an indicial equation $0 = \begin{vmatrix} -(\lambda + k \cdot N + \widehat{f}) & -\widehat{S} \\ h \cdot \widehat{f} & -(\lambda + h \cdot C - h \cdot \widehat{S}) \end{vmatrix}$, λ is obtained as a conjugate complex number $\lambda = \sigma \pm \tau$, where σ and τ are each expressed by:

$$\sigma = -\frac{1}{2}\left[\frac{(h \cdot \widehat{y} + a)^2}{ahC} + \frac{ahC}{h\widehat{y} + a}\right] \tag{9}$$

$$\tau^2 = \frac{1}{4}\left\{\left(\frac{h \cdot \widehat{y}}{a\widehat{S}} C\widehat{f} - \frac{a}{\widehat{f}}\right)^2 - 4h \cdot \widehat{y}\right\}. \tag{10}$$

In Eqs. (7) and (8), in the meantime, B_{ij} is a coefficient obtainable from A_1 and A_2.

With respect to fishing mortality coefficient f, the monthly exploitation rate for month i, represented by f_i, is obtained from Equation (6) as $f_i = \int_0^1 f \cdot dt = \int_0^1 \widehat{f} \cdot dt + \int_0^1 f^* \cdot dt$, and $\int_0^1 \widehat{f} \cdot dt$ can be expressed from Equation (4) in the form of a function of the monthly catch $\widehat{y}_i$ in the case of f and S being in equilibrium, as follows:

$$\begin{aligned}\widehat{f}_i = \int_0^1 \widehat{f} \cdot dt &= \frac{1}{hC}(a + h \cdot \Sigma y) \\ &= \frac{1}{hC}(a + h \cdot \widehat{y}_i)\,.\end{aligned} \tag{11}$$

On the other hand, f_i^* is expressed in the form of an integral of Equation (8) as follows:

$$\begin{aligned} f_i^* &= \int_0^1 f^* \cdot dt \\ &= p\,[B_{21} \sin(\overline{\tau} + \theta) - B_{22} \cos(\overline{\tau} + \theta)] \cdot e^{\overline{\sigma}} - K_f \end{aligned} \tag{12}$$

where $p = \sqrt{\left(\frac{1}{\overline{\sigma}}\right)^2 + \left(\frac{1}{\overline{\tau}}\right)^2}$; $\theta = \tan^{-1}\frac{\overline{\tau}}{\overline{\sigma}}$; $\overline{\sigma}$ and $\overline{\tau}$ are values given by substituting $\widehat{y}_i$ for $\widehat{y}$ in Eqs. (9) and (10), respectively; and K_f indicates an initial value. Consequently, f_i^* can be expressed in the form of a function of $\widehat{y}_i$, and the same is true for S_i^*. If $\overline{\sigma}$ is given in this case by an increasing function for $\widehat{y}_i$ with respect to exploitation rate f_i, it may be said that f_i^* is a variation which stays in continuous amplitude around Equation (11) and whose oscillation increases with increasing $\widehat{y}_i$.

When applying an appropriate method of moving average to the variation of exploitation rate f_i which has the characteristics described above, its oscillation part is smoothed out and $\widetilde{\widetilde{f}}_i \doteqdot \widehat{f}_i$ may be as assumed. If $y_i = (\widehat{f}_i + f_i^*)\ (\widehat{S}_i + S_i^*)$ is assumed for catch quantity y_i and minor terms of an order higher than that of f^* and S^* are omitted, then y_i is expressed by $y_i \doteqdot \widehat{f}_i S_i^* + \widehat{S}_i f_i^* + \widehat{f}_i \widehat{S}_i = \widehat{f}_i S_i^* + \widehat{S}_i f_i^* + \widehat{y}$, and if $y_i = \widehat{y}_i + y_i^*$ is assumed, y_i^* is expressed by $\widehat{f}_i$, $\widehat{S}_i$, f_i^* and S_i^*. After all, it may also be said by similar reasoning described earlier that y_i^*, too, shows an oscillatory variation with respect to $\widehat{y}_i$. Thus, $\widetilde{\widetilde{y}}_i \doteqdot \widehat{y}_i$ may safely be assumed because the oscillation part of this parameter, like that of the former one, can be smoothed out by a method of moving average.

When exploitation rate f_i corresponding to monthly catch quantity y_i is arranged in descending order of y_i, and an appropriate moving average procedure is applied, then the moving average value for the two parameters are roughly assumed to be $\widetilde{\widetilde{y}}_i \doteqdot \widehat{y}_i$ and $\widetilde{\widetilde{f}}_i \doteqdot \widehat{f}_i$, respectively. Using Equation (11), the following approximate expression can be formed:

$$\widetilde{\widetilde{f}}_i \doteqdot \frac{1}{hC}(a + h\widetilde{\widetilde{y}}_i) = \frac{a}{hC} + \frac{1}{C}\widetilde{\widetilde{y}}_i \tag{13}$$

though the approximation is not so good as in the case of f_i and y_i each of which individually corresponds to $\widehat{y}_i$. If $\tau^2 > 0$, f^* and S^* give smooth curves approaching lines which represent an equilibrium, indicating that the equilibrium is stable.

Method of Effect Calculation

Linear regression coefficient C, i.e. the maximum capacity of an artificial reef, will be found from Equation (13) if material can be obtained concerning monthly exploitation rate f_i and monthly catch quantity y_i. (The unit of time interval need not be limited to a single month alone but the most suitable one may be chosen, such as one day, one week, ten days or two months.)

It is possible to calculate by following the change of C as obtained every year, in what way and to what extent an artificial reef takes effect if sufficient material has been collected concerning the fishing operation during several years before and after the setting of the reef. If f_i is unknown, it should be added, relative capacity C' can be obtained by finding, instead of f_i, relative catch effort, e.g. frequency of net hauling or number of boat put into the fishing operation.

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人工魚礁の効果算定に関する理論的考察

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魚礁設置の前と後の漁獲量努力量統計を利用して，人工魚礁の効果算定をする簡単な方法を提案する。

魚礁効果と漁獲係数は下記の微分式で表現できる。

$$\frac{dS}{dt} = kN(C - S) - y, \quad \frac{df}{dt} = a - hCf + hy$$

ここで S は魚礁に現存する資源量，f は漁獲係数，C は魚礁の最大収容能力，N は漁場に来遊する資源の大きさ，y は漁獲量，k は魚礁に遭遇する個体の添加係数，a は最高漁獲係数の傾向を示す定数（$= k \cdot C \cdot f_{\max}$），$h$ は係数である。

S と f が平衡状態の時，$\widehat{f} = a / hC + \widehat{y} / C$ が導かれる。ある時間間隔（例えば，月や週）の漁獲率 f_t と漁獲量 y_t は一つの平衡状態のまわりを振動する曲線となり，移動平均を適用すると

$$\tilde{\tilde{f}}_t = a / hC + \tilde{\tilde{y}}_t / C$$

が成立する。

人工魚礁の設置前と後の数年間の f_t と y_t が得られれば，年ごとに求めた一次回帰係数 C，即ち魚礁の最大収容能力の変化傾向から，効果の表われ方や大きさを把握することができる。