# A DERIVATION OF MATRIC POTENTIAL
# IN FROZEN SOIL

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### Abstract

Matric potential in frozen soil is derived from equilibrium between unfrozen water and ice in frozen saline soil. The matric potential in frozen soil, expressed here in pressure unit, is approximately expressed as a linear function of temperature, ice pressure and osmotic pressure, of which coefficients are 1.23 (MPa/°C) , 1.1 and 1.0, respectively. The direction of water movement is determined by a sign of the difference in the sum of matric plus gravitational potentials between frozen and unfrozen regions. It is predicted from the derived equation that heaving pressure is proportional to freezing temperature and decreases with increasing solute concentration.

Key words: Matric potential · Unfrozen water · Temperature · Solute · Heaving pressure

## I. Introduction

Freezing and frost heaving are of great consequence in cold regions because they bring on several disasters, such as destruction of paved roads, breakdown of buildings, heaving of railways, and severance of plant roots.[1] In addition, the influence of chemicals on the soil freezing phenomenon has recently been concerned from the viewpoint of chemical pollution of the environment.[2-4]

The mechanism of frost heaving has been studied in the past by many investigators.[1-6] As a result of such studies, it is revealed that unfrozen film water exists[5-7] and migrates to ice lens in frozen soils.[1-2] Figure 1 shows a schematic diagram of the unfrozen water in frozen soil. The existence of the unfrozen water is explained in terms of the decrease of chemical potential of water attracted on clay particles.[8-9] The migration of water in unsaturated soils is usually explained in terms of matric potential and hydraulic conductivity. In the unfrozen soils, the matric potential is acceptable because it is related to matric suction which is easy to measure as a pressure by means of tensiometer. In frozen soils, however, it is so difficult to measure directly the pressures of both unfrozen water and ice that we cannot use the normal concept of pressure of water to express the driving force for water movement.

The ice pressure in frozen soil is often regarded as atmospheric pressure for unsaturated soil and overburden/frost heaving pressure for saturated soil. Thereby, a derivation of an equation has been proposed for the thermodynamic equilibrium between ice and water in porous media.[1,10] However, because the derivation has been attempted based on several complicated concepts, the mechanism of unfrozen water movement has not been understood sufficiently. In this paper, a concept of potential of unfrozen water is described in the soil-water systems having solute and overburden pressure, illustrating a model to represent the
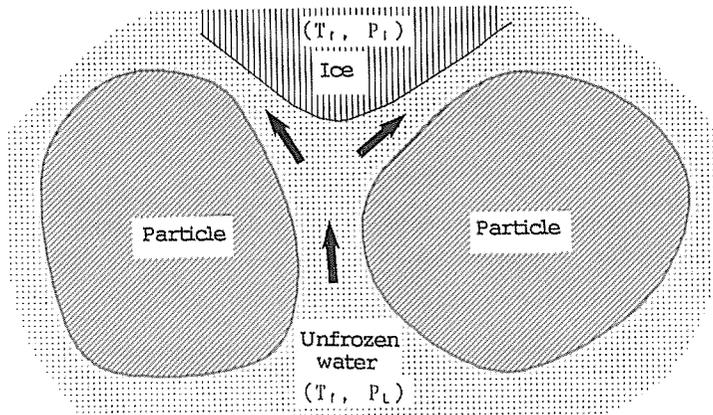
Fig. 1.   A schematic representation of equilibrium between ice and unfrozen water in frozen soil.
$T_f$ is the freezing temperature.   $P_i$ and $P_L$ are the pressures of ice and unfrozen water,
respectively.

relationship between osmotic and overburden pressures.   Then a derivation is given for the matric potential in
frozen soils and an application of the derived equation is presented for various conditions during soil freezing
process.


## II.   Potentials of soil water

The total soil water potential of the constituent water in soil is the amount of useful work per unit quantity of
pure water that must be done by means of externally applied forces to transfer reversibly and isothermally an
infinitesimal amount of water from standard state to soil liquid phase at the point under consideration (Bolt,
1976).[11]   It can be expressed as

$$\psi_t = \psi_g + (\Delta\mu)_T \qquad [J/kg] \tag{1}$$

where $\psi_t$ is the total potential, $\psi_g$ is the gravitational potential, and $(\Delta\mu)_T$ is the chemical potential of soil water at
the same temperature.   The chemical potential of soil water can be expressed as

$$(\Delta\mu)_T = \psi_m + \psi_o \qquad [J/kg] \tag{2}$$

where $\psi_m$ and $\psi_o$ are the potentials of matric and osmotic, respectively.

Although the potential energy per mole, J/mol, is generally used for the unit of the chemical potential, we
use the potential energy per mass, J/kg, in this paper.   The chemical potential of soil water is determined by
that of pure water equilibrated with soil water at the same elevation and temperature by means of a tension plate
assembly, a pressure plate apparatus, psychrometer method, etc.   In the psychrometer method, for example,
the chemical potential of soil water is determined by that of vapor, regarded as an ideal gas, in equilibrium with
the soil water.   For frozen soil the chemical potential of the unfrozen water can be determined by that of the ice
instead of the vapor.

The chemical potential of pure substances is generally expressed as a function of temperature and pressure.
Then we express here the chemical potential of soil water as

$$\Delta\mu = -s_L\Delta T + v_L\Delta P_L \quad [J/kg] \tag{3}$$

where $s_L$ is the specific entorpy of pure water, J/kg/K, $v_L$ is the specific volume of pure water, m$^3$/kg, and $\Delta P_L$ and $\Delta T$ are the pressure, J/m$^3$, and the temperature, K, of soil water relative to the standard state, respectively; that is $\Delta P_L = P_L - P_o$, $\Delta T = T - T_o$, where $P_o$ and $T_o$ are the pressure and the temperature at the standard state (N.T.P.; 0°C, 1 atm). Normally, since water in soil is affected by soil particles and solutes, several terms may be added to the right hand side in equation (3). However, it should be noted here that $\Delta P_L$ includes all terms except temperature because we do not have enough knowledge about interaction between soil and water.

For the isothermal condition, equation (3) becomes

$$(\Delta\mu)_T = v_L\Delta P_L \quad [J/kg] \tag{4}$$

This equation means that the chemical potential is equivalent to the pressure for the isotermal condition. It should be noted here that the pressure used in equations (3) and (4) is not the so-called mechanical pressure but the pressure which would be measured by means of an "ideal" tensiometer equilibrated with soil water. For water in soil solution, thereby, it is useful to regard the pressure as potential energy per unit volume, J/m$^3$, although pressure is normally defined as a force per unit area, N/m$^2$ or Pa.

A concept of the ideal tensiometer may be easy to understand if we use an analogy of osmotic pressure apparatus. In the solution-pure water systems separated with a perfect semipermeable membrane, one must suck the pure water at the osmotic pressure $\Pi$ to keep the mechanical equilibrium in the systems. This means that the structure of water in the solution changes due to mainly hydration and then the potential energy of a water molecule is equal to that of a sucked water molecule. Replacing the solution with soil water, the soil water separated with the perfect semipermeable membrane will also be in equilibrium with the pure water sucked at $\Delta P$ as shown in Fig. 2(A). This equilibrium can be explained by a spring model of water molecules in Fig. 2(B). This figure shows that the soil water, of which potential energy is reduced due to the attractive froce from the soil particle surface, is in mechanical equilibrium with the pure water sucked by a piston.
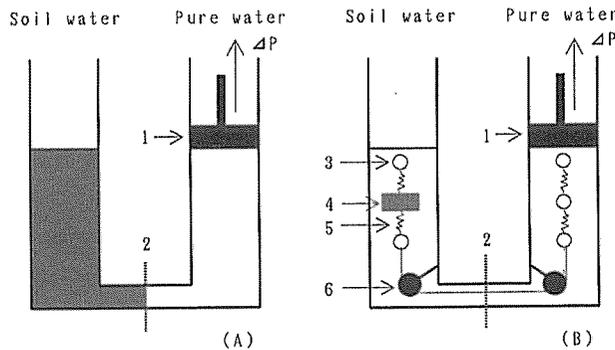


Fig. 2. Equilibrium between soil water and pure water; (A) Soil water separated with a perfect semipermeable membrane (B) Model to represent the equilibrium. 1. Piston  2. Perfect semipermeable membrane  3. Water molecule  4. Soil particle  5. Spring  6. Pulley

Comparing equation (4) with equation (2), the water pressure can be written as

$$\Delta P_L = \Delta P_m - \Pi \tag{5}$$

where $\Delta P_m$ is the matric potential expressed by the pressure unit, which is equal to $\psi_m/v_L$ and usually negative,

$\Pi$ is the osmotic pressure which is equal to $-\psi_o/v_L$ and always positive. For dilute solution, the osmotic pressure[12] is approximately given as

$$\Pi = iC_M RT \tag{6}$$

where $C_M$ is the concentration of soil solution, moles/m$^3$, R is the gas constant, 8.314 J/mole/K, and i is the Van't Hoff coefficient.

## III. EQULIBRIUM BETWEEN UNFROZEN WATER AND ICE IN FROZEN SOIL

Figure 3 illustrates temperature dependence of chemical potentials[13] for liquid water and ice without solute at atmospheric pressure. Pure water exists as ice below 0°C because the chemical potential of ice is lower than that of liquid. On the other hand, it exists as liquid over 0°C because the chemical potential of liquid is lower than that of ice. Since the chemical potentials of liquid and ice are equal at the standard state, ice and liquid can coexist. Then the condition for the equilibrium is

$$\mu_L(T_o, P_o) = \mu_i(T_o, P_o) \tag{7}$$

where $\mu$ is the chemical potential of water, and the subscripts refer to liquid (L) and ice (i).
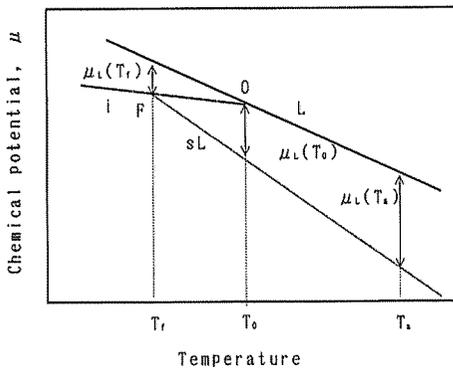


Fig. 3. Chemical potentials of pure water and ice at atmosphere as a function of temperature. i: ice L: Liquid water sL: Soil water O: Normal freezing poing F: Freezing point of soil water.

When ice is in equilibrium with unfrozen water in frozen soil at the temperature of $T_f$, the chemical potentials of liquid and ice are equal;

$$\mu_L(T_f, P_L) = \mu_i(T_f, P_i) \tag{8}$$

where $P_L$ is the pressure of unfrozen water, and $P_i$ is the pressure of ice which is often postulated to be equal to overburden pressure. Using equation (3), the chemical potentials of unfrozen water and ice relative to the standard state can be approximately expressed as

$$\mu_L(T_f, P_L) = \mu_L(T_o, P_o) - s_L \Delta T_f + v_L \Delta P_L \tag{9}$$

$$\mu_i(T_f, P_i) = \mu_i(T_o, P_o) - s_i \Delta T_f + v_i \Delta P_i \tag{10}$$

where $s_i$ and $v_i$ are the specific entropy and the specific volume of ice, respectively. Substituing equations (9) and (10) into equation (8) and using equation (7), then we can get a generalized Clausius-Clapayron equation

$$v_L \Delta P_L = (s_L - s_i) \Delta T_f + v_i \Delta P_i \tag{11}$$

This equation indicates that the chemical potential of unfrozen water can be determined by temperature and pressure.    Equation (11) is rewritten in the pressure unit as

$$\Delta P_L = (\Delta_i^L s / v_L) \Delta T_f + (v_i / v_L) \Delta P_i \tag{12}$$

where $\Delta_i^L s$ is written for the deference in specific entropy between $s_L$ and $s_i$, $s_L - s_i$.    Since $\Delta s = \Delta h / T_o$, in which h is the enthalpy per unit mass, inserting approximate values for the constans of water, $\Delta h = 336 \text{ kJ/kg}$, $v_L = 10^{-3} \text{ m}^3/\text{kg}$, $v_i = 1.1 \times 10^{-3} \text{ m}^3/\text{kg}$, $T_o = 273.15 \text{ K}$, gives the practical equation at the range of temperature near $0°C$:

$$\Delta P_L = 12.3 * \Delta T_f + 1.1 * \Delta P_i \qquad [10^5 \text{ Pa}] \tag{13}$$

This equation means that the pressure of unfrozen water can be expressed as a linear function of temperature and ice pressure.

Strictly speaking, the coefficients of $\Delta T_f$ and $\Delta P_i$ should also be expressed as a function of temperature and pressure because h, $v_L$ and $v_i$ are depend on temperature and pressure.    However, since the temperature dependence of them is usually small, this equation is valid sufficiently near $0°C$ and very useful for evaluating the pressure of the unfrozen water from the temperature and heaving pressure.


## IV.   MATRIC POTENTIAL IN FROZEN SOIL

The matric potential in saline soil is expressed in terms of the pure water potntial and the osmotic potential as

$$\Delta P_m = \Delta P_L + \Pi \tag{14}$$

Substituting equation (13) into equation (14), the matric potential in frozen saline soil can be defined as

$$\Delta P_m = 12.3 * \Delta T_f + 1.1 * \Delta P_i + \Pi \qquad [10^5 \text{ Pa}] \tag{15}$$

This equation means that the matric potential in the soil is expressed approximately as a linear function of temperature, ice pressure and osmotic pressure.    For unsaturated soil freezing without overburden pressure, ice in the soil may be at atmospheric pressure.    Then since $\Delta P_i = 0$, equation (15) becomes

$$\Delta P_m = 12.3 * \Delta T_f + \Pi \qquad [10^5 \text{ Pa}] \tag{16}$$

It is noted in the equation that the matric potential is expressed as a function of temperature and osmotic pressure.    Furthermore, the matric potential is proportional to only freezing point depression for salt free unsaturated soil; that is,

$$\Delta P_m = 12.3 * \Delta T_f \qquad [10^5 \text{ Pa}] \tag{17}$$

The equation suggests that the unfrozen water in the soil at the temperature of $-1°C$ should be equivalent to the pure water sucked mechanically at the pressure of $1.23 \text{ MPa}$ ($12.3 \text{ bar}$).

Using the unit of water head, $cmH_2O$, the logarithm of equation (17) gives the relationship between pF and the freezing point depression, Schofield equation[14]:

$$pF = 4.1 + \log|\Delta T_f| \tag{18}$$

This equation is well known to be valid near $0°C$.    With respect to the equation, Low et al.[5] derived the

modified equation available for the wide range of temperature by considering the temperature dependence of the enthalpy of water.

When the soil water freezes, the thickness of unfrozen film water decreases. As a result, the matric potential in frozen also decreases. If the soil water contains solute, the concentration/the osmotic pressure of the residual unfrozen water will increase as the soil water freezes. The freezing point of soil water is then depressed due to both the decrease of unfrozen water thickness and the increase of osmotic pressure. If the soil water freezes under overburden pressure, the freezing point depression will increase. Therefore, equation (15) is very important as a state equation for connecting the matric potential in frozen soil to temperature and osmotic pressure.

When we conduct a computer simulation for the simultaneous transport of water, heat and solute in freezing unsaturated soil, we will find the temperature important to estimate the unfrozen water and ice contents in the frozen regions. Therefore, it is useful to express the state equation for the temperature. Solving equation (16) for the temperature, the generalized freezing point depression equation yields:

$$\Delta T_f = 0.081 \times 10^{-5} * \Delta P_m - 1.86 * i * C_m \tag{19}$$

where $C_m$ is the concentration of soil solution, moles/kg. The first term of the right hand side is due to the matric potential and the second term is due to the solutes.


## V. SOIL FREEZING AND WATER MOVEMENT

Liquid water content in freezing soil decreases abruptly because of phase change from the liquid to ice. Soil water moves from unfrozen to frozen regions in order to recover the decrease of the liquid water. Using the matric potential in frozen soil defined in this paper, we can deal with the continuous water movement in both frozen and unfrozen regions.

The direction of water movement is determined by a sign of the difference in the sum of matric plus gravitational potentials between frozen and unfrozen regions:

$$\begin{aligned}
\delta P_m &= (\Delta P_m + \rho g Z)_f - (\Delta P_m + \rho g Z)_u \\
&= (\Delta P_m)_f - (\Delta P_m)_u + \rho g (Z_f - Z_u)
\end{aligned} \tag{20}$$

where $\rho$ is the density of liquid water, g is the acceleration of gravity, Z is the elevation, of the location of interest in the soil. Subscripts refer to frozen(f) and unfrozen(u) regions. When $\delta P_m$ is negative, soil water will move from unfrozen to frozen regions.

According to freezing conditions, three aspects are predicted:

(1)    In unsaturated soil freezing, ice grows at atmospheric pressure in soil, and the matric potential in frozen regions is determined by equation (16). Therefore, if $\delta P_m$ is negative, water moves from unfrozen to frozen regions.

(2)    After the void in soil is filled with the ice, the ice pressure increases and the matric potential in frozen soil increases according to equation (15). If $\delta P_m$ keeps negative, water continues to move from unfrozen to frozen regions in spite of the existence of the ice pressure.

(3)    When the ice pressure increases more and $\delta P_m$ becomes zero, the movement of unfrozen water ceases. The ice pressure which stops the water movement to frozen regions is often regarded as maximum heaving

pressure. Takashi et al.[15] have measured the relationship between the heaving pressure and freezing temperature in a saturated soil of which unfozen end is connected to a water reservoir.

For water in the reservoir $\Delta P_L$ is zero because there is no soil in it. Therefore, equation (14) becomes:

$$(\Delta P_m)_u = \Pi_u \tag{21}$$

Substituting equations (15) and (21) into equation (20), and using the condition of $\delta P_m = 0$, then we can derive an equation to express the relationship between the ice pressure (heaving pressure) and the temperature related to solutes.

$$\Delta P_i = -11.2 * \Delta T_f - 0.9 * \delta \Pi - 0.09 * \delta Z \quad [10^5 \, Pa] \tag{22}$$

where $\delta$ is written for the difference between unfrozen water in frozen soil and bulk water in the reservoir. The third term of the right hand side will normally be negligible. For the experiment by Takashi et al., $\delta \Pi$ will be zero because they have used a salt free water. Therefore, equation (22) becomes modified Clausius-Clapayron equation:

$$\Delta P_i = -11.2 * \Delta T_f \quad [10^5 \, Pa] \tag{23}$$

The coefficient of $\Delta T_f$, $-11.2$, is known to agree well with the experimental result. Anyway, it should be noted in equation (22) that the equation predicts that the heaving pressure will decrease in frozen saturated soils containing solutes.

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# 凍土内のマトリックポテンシャルの誘導

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　塩分を含む凍土のマトリックポテンシャルを不凍水と氷の平衡から誘導した。圧力単位で表示した凍土のマトリックポテンシャルは近似的に温度，氷圧および浸透圧の線形和で表現される。このとき，それぞれの係数は 1.23 (MPa/℃)，1.1および1.0である。土の凍結過程の水分移動の方向はマトリックと重力ポテンシャル和の凍結部および未凍結部の差の符号によって決定される。本論文の誘導式から，凍上圧は凍結温度に比例し溶質濃度が増加すると減少することが予測できる。