(**Review**)

Application of Contact Theory to the Bean Impact Problem

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1. Introduction

Mechanical damage to beans caused by harvesting, cleaning and handling equipment has become an aggravating and costly problem in recent years. It is necessary to minimize mechanical damage by developing new machines and new processes that allow the bean to retain its wholeness when processed. Mechanical damage of beans is usually caused when they are threshed by a threshing cylinder and transported through separating and processing systems. The main cause of bean damage seems to be the impacting of beans against various elements of harvesting systems. It is the purpose of this paper: 1) to review the contact theory used for the design of fruit harvesting equipment for minimum bruising by Horsfield,[2] 2) to apply it to the evaluation of (a) the importance of material properties of the bean, (b) the impact device and (c) the impact velocity on bean damage.

2. Review of contact theory for application to beans

According to the theory for two impacting spheres given by Timoshenko[9] and also Goldsmith[1], who extended Hertz's contact theory, the approach α_1 of the spheres at the point of maximum compression can be predicted by

$$\alpha_1 = \left(\frac{5}{4} \frac{V^2}{n n_1} \right)^{2/5} \tag{1}$$

where V = relative velocity of approach of both spheres

$$n_1 = \frac{m_1 + m_2}{m_1 m_2}$$

$$n = \sqrt{\frac{16}{9\pi^2} \frac{R_1 R_2}{(K_1+K_2)^2 \ (R_1+R_2)}}$$

m_1 = mass of first sphere
m_2 = mass of second sphere
R_1 = radius of first sphere
R_2 = radius of second sphere

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$$K_1=\frac{1-\nu_1^2}{\pi E_1}$$

$$K_2=\frac{1-\nu_2^2}{\pi E_2}$$

ν_1 =Poisson's ratio of first sphere
ν_2 =Poisson's ratio of second sphere
E_1 =modulus of elasticity of first sphere
E_2 =modulus of elasticity of second sphere

Timoshenko[9] also derived the expression of the approach α of the spheres two in terms of the pressure between the bodies and radius of the contact area, as follows:

$$\alpha= (K_1+K_2)\ P_0\frac{\pi^2 a}{2} \tag{2}$$

where P_0 =pressure at the center of the contact area.
a=radius of the contact surface of the two spheres.
Eq.(2) can also be expressed as follows[9]

$$a=(K_1+K_2)\frac{\pi^2}{2}P_0\frac{R_1 R_2}{R_1+R_2} \tag{3}$$

Solving eq.(3) for P_0, substituting into eq.(2), and solving for α results in

$$\alpha=a^2\ \frac{(R_1+R_2)}{R_1 R_2} \tag{4}$$

When afalling sphere mass m_1 impact a surface of mass m_2, and m_2 is very large compared with m_1, then the impact velocity V and the mass term n in eq.(1) can be replaced by the drop height h and the sphere weight W using the following identities:

$$V^2=2gh \tag{5}$$

and

$$W=m_1 g \tag{6}$$

where g = acceleration due to gravity (in/sec^2)
From eqs. (5) and (6),

$$V^2=\frac{2Wh}{m_1} \tag{7}$$

Since

$$n_1=\frac{m_1+m_2}{m_1\ m_2}=\frac{m_1/m_2+1}{m_1}$$

for $m_2 \gg m_1$

$$n_1=\frac{1}{m_1} \tag{8}$$

Assuming Poisson's ratio to be 0.49 for both sphere and surface, eq.(1) can be substituted into eq.(4). Then the following equation for the radius of the contact area, using units of lb and in, can de obtained.

$$a = 1.077\ (W\ h)^{1/5} \left(\frac{E_1 + E_2}{E_1 E_2} \right)^{1/5} \left(\frac{R_1 R_2}{R_1 + R_2} \right)^{2/5}$$

$$= 1.077\ \left(\frac{1}{2} m_1 V^2 \right)^{1/5} \left(\frac{E_1 + E_2}{E_1 E_2} \right)^{1/5} \left(\frac{R_1 R_2}{R_1 + R_2} \right)^{2/5} \qquad (9)$$

Substituting eq. (9) into eq. (2) and using eq. (4) yields

$$p_0 = 0.899\ \left(\frac{1}{2} m_1 V^2 \right)^{1/5} \left(\frac{E_1 E_2}{E_1 + E_2} \right)^{4/5} \left(\frac{R_1 + R_2}{R_1 R_2} \right)^{3/5} \qquad (10)$$

where the units are in lb/in^2.
In addition to giving the maximum compressive stress, eq. (10) can be used to determine the maximum internal shear stress τ_{max} which occurs at the boundary of the circle of contact by using the following relationship[1]

$$\tau_{max} = 0.27 P_0 \qquad (11)$$

Before applying the above equations to bean impact problem, the assumptions used in the derivation must be considered.

The assumptions made by Hertz are the following:

(1) The material of the contacting bodies is homogenous.
(2) The loads applied are static.
(3) Hooke's law holds.
(4) Contacting stresses vanish at the opposite ends of the body (semi-infinite body).
(5) The radius of curvature of the contacting solid is very large compared with the radius of area of contact.
(6) The surfaces of the contacting bodies are sufficiently smooth that tangential forces are eliminated.

The assumption regarding the homogeneity of the material is a serious restriction with beans if they are impacted from the bottom, top or front because they have two cotyledons. The assumption can be made however with the case of side impact neglecting the effect of thin seed coat.

According to Pao[8] Hertz's contact theory is valid only if the impact is of very long duration in comparison with the time required for the wave propagation. Mohsenin[8] concluded that "apparently during the deformation process there is a sufficient time for elastic waves to travel to and fro several times before they are dissipated throughout the colliding bodies."

Regarding assumption (3) Lee and Radoh[4] have shown that, with loading times of less than one fourth of the relaxation time (an indication of the degree of viscoelasticity), biological materials can be considered elastic. For bean impact situations this condition exists.

The assumption regarding the ratio of the radius of contact area to the surface radius is also valid since the bean has relatively high elastic modulus resulting in a small radius of the contact area. The rest of the assumptions regarding contacting stress vanishment and surface smoothness are not critical for beans.

For impacting beans with a flat, rigid surface:

$R_2 \gg R_1$ and $E_2 \gg E_1$, then,

$$\text{for } R_2 \gg R_1 \quad \frac{R_1 R_2}{R_1+R_2} = \frac{R_1}{R_1/R_2+1} = R_1$$

$$\text{for } E_2 \gg E_1 \quad \frac{E_1+E_2}{E_1 E_2} = \frac{E_1/E_2+1}{E_1} = \frac{1}{E_1}$$

Eq.(9) can be solved for the modulus of elasticity of the sphere as follows:

$$\begin{aligned} E_1 &= 1.449 \frac{WhR_1^2}{a^5} \\ &= 0.7245 \frac{m_1 V^2 R_1^2}{a^5} \end{aligned} \tag{12}$$

where R_1 is the radius of the sphere (bean). The modulus therefore can be determined for an impact situation, if the energy Wh or $\frac{1}{2}m_1V^2$ is known, the surface radius at the point of contact is known, and the contact-area radius is known. Once the radius is determined, the maximum shear stress which occurs during an impact can be found using eqs. (10) and (11).

Because local surfaces of beans have to be described as ellipsoidal, the equations for the mobulus and maximum shear stress have to be modified by using the following relations derived by Timoshenko [9]

$$R = \frac{2R_a R_b}{R_a+R_b}, \quad a = \frac{2a_a a_b}{a_a+a_b}$$

where subscripts a and b respectively refer to the major and minor radii.

3. Applicaton of the theory for evaluation of bean damage

From eqs. (10) and (11) maximum pressure and maximum shear stress are proportional to an energy term, a modulus term and a radius term as follows:

$$\tau_{max} = 0.27 P_0 \propto (m_1 V^2)^{1/5}, \left(\frac{E_1 E_2}{E_1+E_2}\right)^{4/5}, \text{ and} \left(\frac{R_1+R_2}{R_1 R_2}\right)^{3/5} \tag{13}$$

For bean harvest and handling systems eq. (13) shows that for preventing mechanical damage during impact it is more important to use a low modulus material for impacting parts than it is to reduce impact energy. It is apparent that, for a threshing cylinder, elastic modulus E_2 of the cylinder spikes has more importance than the cylinder speed for reducing mechanical damage to beans. The radius R_1 of the bean is fixed. It is then necessary to avoid impact on hard surfaces with radii much smaller than those of the beans. Eq. (13) also shows the difficulty of using impact energy or impact velocity to predict damage to beans during impact. A very small change of dean modulus E_1 has a greater effect on maximum stress or internal shear than does a change in impact velocity.

From eqs. (10) and (11) it is also seen that once the allowable stress or shear strength, elastic modulus, radius and weight of the bean are known, the eq. (10) can be used to evaluate impact velocities, surface moduli and radii which will prevent bean damage caused by different parts of the harvesting and the handling systems. The effect of bean weight is less critical compared to the other parameters discussed above.

Assuming that large and small beans have the same elastic modulus and allowable stress.

and the weight is proportional to R_1^3, the maximum allowable impact velocity without damage can be evaluated as follows:
From eq. (10)

$$V \propto \left[\left(\frac{R_1 R_2}{R_1 + R_2} \right)^3 \left(\frac{1}{R_1} \right)^3 \right]^{1/2} = \left(\frac{R_2}{R_1 + R_2} \right)^{3/2}$$

If the surface radius R_2 is large compared to the bean radius R_1 then,

$$\frac{R_2}{R_1 + R_2} = \frac{1}{R_1/R_2 + 1} = 1$$

Thus the bean size has little effect on damage susceptibility. However if the beans are small, they are less likely to be damaged.

4. Some comments on the contact theory

1) It is assumed in this paper that the poisson's ratio is 0.49 for both sphere and the impact surface to derive the eqs. (9) and (10). But this may not be the case which actually exists for biological materials and impact surface materials. Both values probably vary and the Posisson's ratio of biological products may likely to be dependent on the moisture content. This situation allows the application of the above equations to the case only if Poisson's ratio and the moisture content of the biological material are known.

Narayan[6] obtained the elastic modulus of navy beans by using buckling theory considering a bean to be a beam with variable section area. To see the applicability of this contact theory to bean impact situation, eq. (9) has been used to examine how close the value of contact area calculated from the available data is to the values which can be assumed to be reasonable. The following values obtained by Narayan are used to evaluate the area of contact between a bean and an impacting surface of flat steel.

$E_1 = 1 \times 10^4$ psi (for navy bean of high moisture)
$E_2 = 30 \times 10^6$ psi (for steel)
$V = 2000$fpm$= 400$ in/sec (impact velocity)
$W = 4 \times 10^{-4}$ lb (bean weight)

$$\left. \begin{aligned} R_{1a} &= \frac{3}{32}\text{in} \\ R_{1b} &= \frac{1}{4}\text{in} \end{aligned} \right\} \text{(for side impact)}$$

$$R_1 = \frac{2R_{1a}R_{1b}}{R_{1a} + R_{1b}} = \frac{2 \cdot \frac{3}{32} \cdot \frac{1}{4}}{\frac{3}{32} + \frac{1}{4}} = 0.167 \text{ in}$$

$$m_1 = \frac{W}{g} = \frac{4 \times 10^{-4}\text{ lb}}{388 \text{ in/sec}^2} = 1.03 \times 10^{-6} \text{ lb}\cdot\text{sec}^2/\text{in}$$

$R_2 = \infty$ (flat surface)

Putting these values into eq. (9) we find

$$a=1.077\left(\frac{1}{2}\times 1.03\times 10^{-6}\times 400^2\right)^{1/5}\left(\frac{10^4+30\times 10^6}{10^4\times 30\times 10^6}\right)^{1/5}\times\left(\frac{0.167\times\infty}{0.167+\infty}\right)^{2/5}$$

$$=0.0506 \text{ in}$$

Also for beans of $E_1 = 2\times 10^4$ psi (for navy beans of low moisture)

$$a=0.0441 \text{ in}$$

These values for the contact area seem reasonable. The application of contact theory to beans therefore seems to be a promising approach to the analysis of the bean impact problem. By knowing specific values of Poisson's ratio for beans, more accurate evaluation of the parameters such as a, E_1, P and τ_{max} can be made from the eqs. (9), (10), (11) and (12).

2) For the use of eq. (12) to evaluate the elastic modulus of beans, one problem is that elastic modulus is inversely proportional to the fifth power of the radius of contact area. This may lead to erroneous results unless adequate precautions are taken to measure the contact area. The comparison of the results obtained from contact theory with the results of other theories, will be of interrest and also be useful for investigating the applicabilities and the limitations of the individual theories to specific conditions of beans.

3) The eq. (11) expressing the relationship between maximum pressure and maximum shear stress is derived for the case of a steel sphere[1] pressed against a plane steel surface. This relationship may not be applicable for the case of biological material. . A modification of the equation may be needed for a specific analysis of been damage mechanism.

5. Explanation of bean damage by contact theory

An explanation can be made on bean impact damage which usually increases at a low moisture content.[3][6][7] The elastic modulus increases with the decrease of moisture content.[6] According to eq. (13) the maximum pressure and maximum shear stress increase in proportion with the elastic modulus to the four fifth power of the bean, if the impact surface has a very large value of elastic modulus since

$$\text{for}\quad E_2 \gg E_1 \qquad \frac{E_1 E_2}{E_1+E_2}=\frac{E_1}{E_1/E_2+1}=E_1$$

Therefore the maximum stress occuring in the bean becomes high at low moisture contents, resulting in the failure of the bean material.

The data available on the effect of bean impact velocity and bean weight upon the bean damage are limitted. Some experimental results were obtained by Hoki[3] indicating considerable effects upon the bean damage. It is difficult to make any quantitative evaluation of the results. The effect of bean weight however seems to be more significant than predicted from eq. (13), since this is only one fifth power of the weight increase. Further investigation will be necessary on this subject.

The effect of impact velocity is considerable at the high velocity range. Quantitative evaluating the contact theoty is again difficult to make, but the amount of damage should increase with the impact velocity to the two fifth power. Detailed examination of bean damage should be made for evaluating the applicability of contact theory to impact velocity and damage analysis.

Summary and conclusions

The Hertz's contact theory applied to fruit bruising has been critically reviewed. Its application to the been impact problem has been considered. By using available data on bean properties, applicability of contact theory to bean impact has been investigated.

Conclusions derived are as follows:

1) The application of contact theory for evaluating relative importance of mechanical properties of beans seems to be promising.
2) The important factors affecting bean bamage are the elastic moduli of beans and impact surfaces, radii of beans and impact surfaces, and impact velocities. The effect of bean weight and size is less critical compared to the other factors. However this also depends upon the elastic modulus of impacting surface.
3) With a reasonable assumption of poisson's ratio for the beans it is possible to evaluate the elastic modulus by using contact theory. The comparison of the results obtained by the contact theory approach with the results obtained by other methods such as compression tests, bending tests or vibration tests, will be useful for better understanding of the applicability and the limitation of the individual methods.
4) To evaluate the elastic modulus of a bean by contact theory, accurate measurements of the radius of the contact area and the bean surface radius are necessary. Suitable measuring method of those parameters should be established.
5) It should be possible to predict maximum pressure or shear stress occuring in the bean during impact by using contact theory.

References

1. Goldsmith, W. Impact: the theory and physical behavior of colliding bodies. Edward Arnold Publishers, Ltd. London, 1960.
2. Horsfield, B. C. et. al. Application of theory of elasticity to the design of fruit harvesting and handling equipment for minimum bruising. ASAE paper No. 70-811. 1970.
3. Hoki, M. and L. K. Pickett. Factors affecting mechanical damage of navy beans. Trans. ASAE 16(6): 1154-1157, 1973.
4. Lee, E. H. and J. R. M. Radoh. The contact problem for viscoelastic bodies. Trans. ASME. Jonrnal of Applied Mechanics 27(9): 438-444, 1960.
5. Mohsenin, N. N. Physical properties of plant and animal materials. Vol. I. Gordon and Beach Science Publishers, New York, 1970.
6. Narayan, C. V. Mechanical checking of navy beans. Ph. D. thesis, MSU. 1969.
7. Perry, J, S. Mechanical damage to pea beans as affected by moisture, temperature, and impact loading. Ph. D. thesis, MSU, 1959.
8. Pao, Y. H. Extension of the Hertz theory of impact to the viscoelastic case. Journal of Applied physics 26(9) 1088, 1955.
9. Timoshenko, S. P. and J. M. Goodier. Theory of elasticity. McGrow Hill, New York, 1951.

摘　　要

豆の衝撃に対する接触理論の応用

法貴　誠

すでに果物の損傷に対して用いられているヘルツの接触理論につき，豆の衝撃問題への応用を考察した。豆に関する既知の物性データを用い豆の衝撃問題の若干の理論的解析を行い以下の結論を得た。

1）豆の力学的特性の重要性を評価するため，接触理論を適用することが可能であると思われる。

2）豆の損傷に対して影響をおよぼす因子としては，豆および衝撃面の弾性係数，豆および衝撃面の曲率半径および衝撃速度がある。豆の重量と大きさは上記の因子と比べ影響は少ないと思われるが、これはまた衝撃面の弾性係数に左右される。

3）豆のポアソン比を仮定することにより接触理論を用いて弾性係数を求めることが可能と考えられる。この方法と他の各種方法により求めた弾性係数との比較を行なうことは接触理論の適用性を明らかとする上で意味があると考えられる。

4）接触理論のより有効な適用を計るためには接触面半径の正確な測定方法を見出すことが必要であると思われる。