Identification of Elastic Parameters for Laminated Circular Cylindrical Shells*
(Comparison of Numerical and Experimental Results)

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An inverse analysis method has already been proposed by one of the authors to identify elastic parameters of laminated composite materials using the FEM eigenvalue analysis. The purpose of this study is to apply the proposed method to a laminated circular cylindrical shell and to compare the estimated elastic parameters of the lamina with the obtained experimental ones. First, by applying the experimental modal analysis technique to a laminated circular cylindrical shell with free boundary conditions, natural frequencies and mode shapes are obtained. Next, by considering the obtained natural frequencies and mode shapes, the elastic parameters for the lamina of the shell are identified. On the other hand, the elastic parameters of the lamina are obtained experimentally. These experimental elastic parameters were compared, and found to agree well, with the elastic parameters identified using the proposed method.

Key Words: Identification, Vibration of Continuous System, Composite Material, Free Vibration, Finite Element Method, Elastic Parameters

1. Introduction

Since composite materials such as fiber reinforced plastic (FRP) have high specific strength and high specific modulus, they have been used in many structural applications, aerospace structures, and sport goods. It is therefore very important to make clear the dynamical properties of the laminated composites for the design and the structural analysis. Especially, elastic parameters are essential for the structural analysis. The elastic parameters of laminated composite materials, however, are difficult to determine by either theoretical or experimental approaches because of their anisotropy.

Deobald determined two Young's moduli, the in-plane shear modulus, and a Poisson ratio for Aluminum plate and Graphite epoxy plate by using the natural frequencies measured by an impulse technique[1]. The bending stiffness parameters from the vibrational characteristic of the symmetrically laminated plate were estimated by Fukunaga[2]. Saito et al. identified the equivalent elastic parameters of honeycomb sandwich panels[3]. Qian et al. presented a method for identifying elastic and damping properties of composite laminates by using measured complex modal parameters[4]. Ayorinde et al. improved the identification method for the four independent elastic constants of an orthotropic plate by using a resonance data[5]. A direct method of determining the six flexural stiffnesses of thin anisotropic plates was presented[6]. On the other hand, one of the authors has already proposed an inverse analysis method to identify the elastic parameters for laminated composite materials using the FEM eigenvalue analysis and the sensitivity analysis[7],[8]. It is an advantage of this method that one can obtain non-destructively the elastic parameters of the products made of composite materials. Also, the proposed inverse analysis method was applied to a symmetrically and an antisymmetrically laminated square plates. From the comparison of the identified elastic parameters of the lamina and experimental ones, one

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can see the good agreements[9]. As mentioned above, one can find the literatures in respect to a plate, but there are few reports of the identified elastic parameters for isotropic circular cylindrical shells and laminated circular cylindrical shells.

For the above reason, in this paper, the proposed method is applied to an angle-ply laminated circular cylindrical shell and a cross-ply laminated circular cylindrical shell. The proposed method mainly consists of the FEM eigenvalue analysis and the nonlinear optimization method considering the relation between elastic parameters and natural frequencies as a nonlinear system. First, by applying the experimental modal analysis technique to the angle-ply and cross-ply laminated circular cylindrical shells with free boundary conditions, natural frequencies and mode shapes are obtained. Secondly, from the obtained natural frequencies and mode shapes, the elastic parameters of the lamina are identified. Next, to justify the identified elastic parameters, the elastic parameters of the lamina are obtained experimentally. A good agreement is observed from a comparison of experimental and identified elastic parameters of the lamina. Finally, in order to confirm the identified elastic parameters, it is shown that the estimated natural frequencies and mode shapes of the laminated circular cylindrical shells by using these identified elastic parameters.

2. Identification Method

2.1 Model of laminated composites

Let's consider a shell element for the laminated composites as shown in Fig. 1. In Fig. 1, the rectangular coordinate axes, x and y, are taken along the edges and the z axis is normal to the x-y plane. Furthermore, the coordinate axes L and T are parallel to the principal elastic axes of the composite material and the V axis is normal to the L-T plane. The relation between generalized stress and strain of a finite shell element model for a laminated composite using first-order shear deformation theory (FSDT) is

$$\begin{Bmatrix}\sigma\\ \mu\\ \tau\end{Bmatrix}=\boldsymbol{D}_e\begin{Bmatrix}\varepsilon\\ \kappa\\ \gamma\end{Bmatrix}=\begin{bmatrix}\boldsymbol{D}_p & \boldsymbol{D}_c & 0\\ \boldsymbol{D}_c & \boldsymbol{D}_b & 0\\ 0 & 0 & \boldsymbol{D}_s\end{bmatrix}_e\begin{Bmatrix}\varepsilon\\ \kappa\\ \gamma\end{Bmatrix} \tag{1}$$

where σ and ε are in-plane stress and strain vectors, μ and κ are bending moment and curvature vectors, τ and γ express shear stress and strain vectors, respectively. $\boldsymbol{D}_p$, $\boldsymbol{D}_b$, $\boldsymbol{D}_c$, and $\boldsymbol{D}_s$ are the in-plane, bending, coupling, and transverse shear stiffness matrices given by Eq.(2).

$$\left.\begin{aligned}\boldsymbol{D}_p&=\sum_{i=1}^{NTL}\boldsymbol{d}_{ip}(h_i-h_{i-1})\\ \boldsymbol{D}_b&=\frac{1}{3}\sum_{i=1}^{NTL}\boldsymbol{d}_{ip}(h_i^3-h_{i-1}^3)\\ \boldsymbol{D}_c&=\frac{1}{2}\sum_{i=1}^{NTL}\boldsymbol{d}_{ip}(h_i^2-h_{i-1}^2)\\ \boldsymbol{D}_s&=k\sum_{i=1}^{NTL}\boldsymbol{d}_{is}(h_i-h_{i-1})\end{aligned}\right\} \tag{2}$$

where h_{i-1} is the vectorial distance from the mid-plane to the lower surface of the ith lamina, hence, the thickness of the lamina is h_i-h_{i-1}. k is the shear correction coefficient (5/6). NTL is the total number of laminas of the laminated composite. $\boldsymbol{d}_{ip}$ and $\boldsymbol{d}_{is}$ are the in-plane and the transverse shear stiffness matrices of the ith lamina, as follows:

$$\boldsymbol{d}_{ip}=\boldsymbol{T}_i^T\begin{bmatrix}\dfrac{E_L^2}{E_L-E_T\nu_{LT}^2} & \dfrac{E_LE_T\nu_{LT}}{E_L-E_T\nu_{LT}^2} & 0\\ \dfrac{E_LE_T\nu_{LT}}{E_L-E_T\nu_{LT}^2} & \dfrac{E_LE_T}{E_L-E_T\nu_{LT}^2} & 0\\ 0 & 0 & G_{LT}\end{bmatrix}_i\boldsymbol{T}_i \tag{3}$$

$$\boldsymbol{d}_{is}=\boldsymbol{T}_i^T\begin{bmatrix}G_{TV} & 0\\ 0 & G_{VL}\end{bmatrix}_i\boldsymbol{T}_i \tag{4}$$

where E_L and E_T are the elastic moduli in the direction of the parallel and normal to the fiber, respectively. G_{TV}, G_{VL}, and G_{LT} are shear moduli. ν_{LT} is Poisson's ratio. $\boldsymbol{T}_i$ is a transfer matrix of the ith lamina from the material principle directions to the elemental coordinate axes with fiber orientation angle θ (see Fig. 1).

The system stiffness matrix $\boldsymbol{K}$ and system mass matrix $\boldsymbol{M}$ of a laminated shell structure are calculated by

$$\boldsymbol{K}=\sum_{e=1}^{NTE}\boldsymbol{L}_e^T\int_S\boldsymbol{B}_e^T\boldsymbol{D}_e\boldsymbol{B}_e dS\boldsymbol{L}_e \tag{5}$$

$$\boldsymbol{M}=\sum_{e=1}^{NTE}\boldsymbol{L}_e^T\int_S\boldsymbol{N}_e^T\rho_e\boldsymbol{N}_e dS\boldsymbol{L}_e \tag{6}$$

where $\boldsymbol{L}_e$ is a transfer matrix of an element from the elemental coordinate axes to the global coordinate axes and NTE is the total number of elements. ρ_e is the density of an element. The strain-displacement relationship matrix $\boldsymbol{B}_e$ and the shape function $\boldsymbol{N}_e$ are

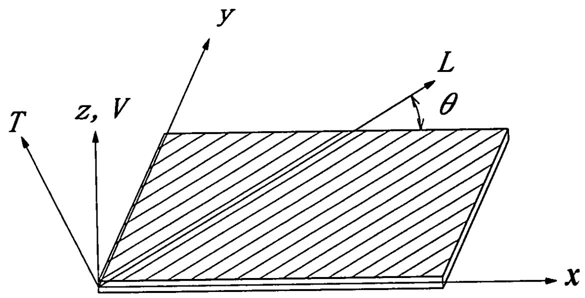

Fig. 1 Coordinate system of a shell element for laminated composites

written as

$$B_e=\begin{Bmatrix} B_p \\ B_b \\ B_s \end{Bmatrix}_e \tag{7}$$

$$N_e=\begin{Bmatrix} N_p \\ N_b \\ N_s \end{Bmatrix}_e \tag{8}$$

The eigenvalue problem of a laminated shell structure is expressed as

$$K\delta_n=\omega_n^2 M\delta_n \tag{9}$$

where δ_n is the nth eigenvector and ω_n is the nth natural circular frequency. The natural frequency f_n of the nth mode of a laminated shell structure is estimated as follows,

$$f_n=\frac{\omega_n}{2\pi} \tag{10}$$

2.2 Nonlinear optimization method

Considering the relationship between natural frequencies and elastic parameters as a nonlinear system, the quasi-Newton method can be used for identifying the elastic parameters of the material principle direction. We define the error function as the difference between the nth vibrational properties f_{En} measured by the excitation test and calculated ones $f_n(\boldsymbol{x})$ by the eigenvalue analysis, as follows:

$$g_n(\boldsymbol{x})=f_{En}-f_n(\boldsymbol{x}) \tag{11}$$

Then, the identification is considered as a nonlinear optimization problem to find a solution $\boldsymbol{x}$ that minimizes the error norm $\Phi(\boldsymbol{x})$.

$$\Phi(\boldsymbol{x})=\frac{1}{2}\sum_{n=1}^{NTM} g_n(\boldsymbol{x})^2 \tag{12}$$

where NTM is the total number of modes.

The quasi-Newton method takes an initial approximation $\boldsymbol{x}_0$, and attempts to improve $\boldsymbol{x}_0$ by the iteration formula using a search direction vector $\boldsymbol{d}$ and a step size parameter λ.

$$\boldsymbol{x}_{k+1}=\boldsymbol{x}_k+\lambda_k \boldsymbol{d}_k \tag{13}$$

A step size parameter λ is chosen by the line searcher algorithm, and a search direction vector $\boldsymbol{d}$ can be given as a solution of the equation followed by

$$\boldsymbol{H}_k\boldsymbol{d}_k=-\nabla\Phi(\boldsymbol{x}_k)=-\boldsymbol{J}^T(\boldsymbol{x}_k)\boldsymbol{g}(\boldsymbol{x}_k) \tag{14}$$

where $\boldsymbol{H}$ and $\boldsymbol{J}$ are the Hessian and Jacobian matrices of error function $\boldsymbol{g}(\boldsymbol{x}_k)$, respectively.

Figure 2 shows the flow chart of the identification program. First, the data about the geometrical configuration, initial material properties of the specimen, and natural frequencies obtained by excitation test are given. Secondly, to calculate the natural frequencies and mode shapes, the eigenvalue analysis is carried out with the initial parameters. Next, the error function $\boldsymbol{g}(\boldsymbol{x})$ is estimated by the difference of the natural frequencies between the eigenvalue analysis and experiment. In this phase, since the order of the modes may be replaced by the ratio of the elastic moduli of anisotropic materials, the order of the natural modes is investigated by MAC (Modal Assurance Criterion). After that, elastic parameters are identified by the quasi-Newton method. Finally, if the calculated natural frequencies are converged into the experimental ones, the identification program is terminated.

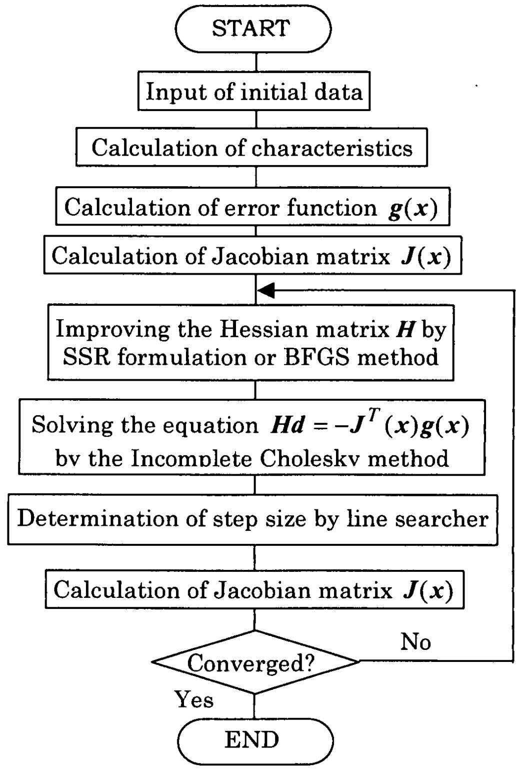

Fig. 2 Flow chart of identification program

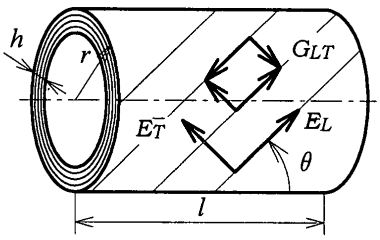

Fig. 3 Laminated circular cylindrical shell

3. Comparison of Identified and Experimental Elastic Parameters

To identify the elastic parameters for the lamina of the laminated circular cylindrical shells with free boundary conditions, experimental studies were carried out. The configuration of the circular cylindrical shell is shown in Fig. 3 and the size of the shell is presented in Table 1, where r is the outside radius. Also, the outside radius r and the thickness h are the average values. The stacking sequence of the angle-ply laminated circular cylindrical shell is [$30°_2$/$-30°_2$/

Table 1 Size of laminated circular cylindrical shell

Type of shell	Length l [mm]	Radius r [mm]	Thickness h [mm]
Angle-ply laminated shell	253 203 151	51.56	1.64
Cross-ply laminated shell	253 203 153	51.57	1.63

Table 2 Number of reference points in experimentation

Type of shell	Length l [mm]	In axial direction	In circumferential direction	Total number
Angle-ply laminated shell	253	11	16	176
	203	11	16	176
	151	9	16	144
Cross-ply laminated shell	253	13	16	208
	203	11	16	176
	153	9	16	144

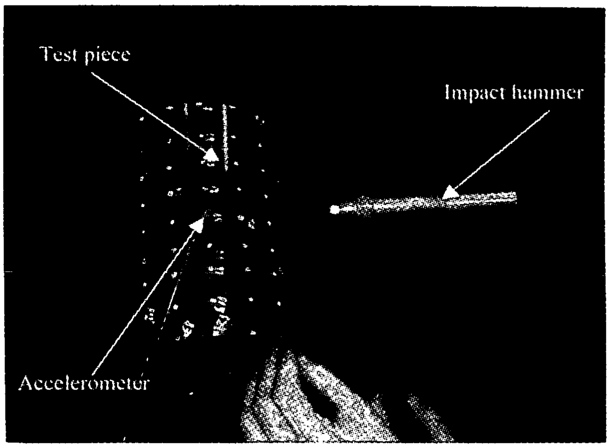

Fig. 4 Measurement of natural frequencies and mode shapes by experimental modal analysis technique

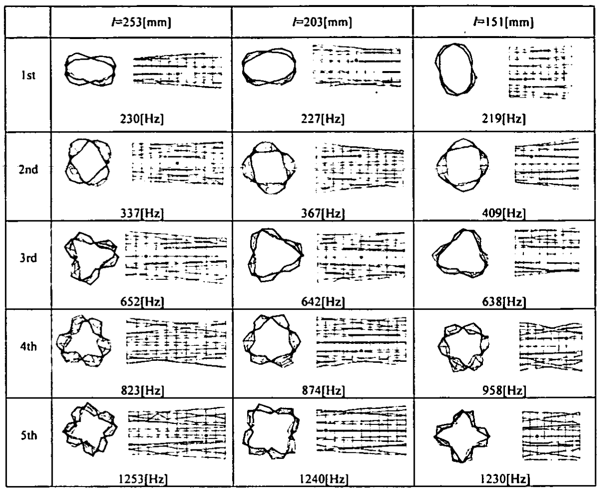

Fig. 5 Experimental natural frequencies and mode shapes of angle-ply laminated circular cylindrical shells ; $[30°_2/-30°_2/-30°_2/30°_2]$

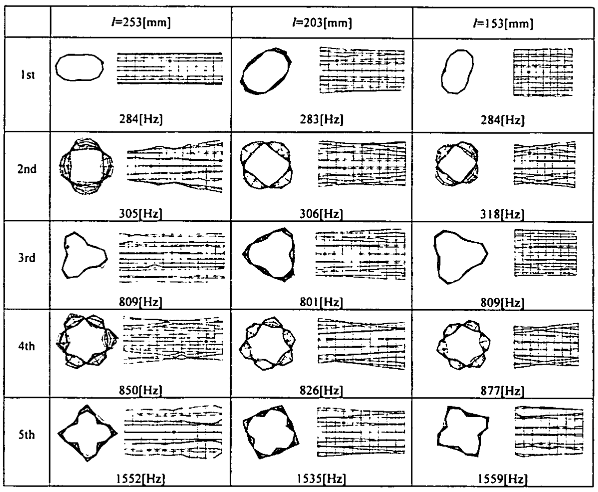

Fig. 6 Experimental natural frequencies and mode shapes of cross-ply laminated circular cylindrical shells ; $[0°_2/90°_2/90°_2/0°_2]$

$-30°_2/30°_2]$. And the stacking sequence of the cross-ply laminated circular cylindrical shell is $[0°_2/90°_2/90°_2/0°_2]$. Each layer material, that is lamina, is a carbon fiber reinforced plastic (CFRP). The laminas of these shells are made of the same prepreg sheet. Therefore, the elastic moduli for the lamina of the each circular cylindrical shell agree, and also the density of these shells is 1495 [kg/m^3].

3.1 Natural frequency and mode shape

To satisfy the free boundary conditions, each laminated circular cylindrical shell was hung from the ceiling by a fine string. As shown in Table 2, each shell was divided into many reference points. To measure the transfer function (accelerance), an accelerometer was attached to one reference point and then all reference points were impacted by an impulse hammer (See Fig. 4). The mass of the accelerometer is 0.48 [g]. From the obtained transfer function, the natural frequencies and mode shapes of the laminated circular cylindrical shells were estimated by applying the experimental modal analysis technique. Figures 5 and 6 show the experimentally obtained natural frequencies and mode shapes of the each laminated circular cylindrical shell. In these figures, the notation "●" represents the location of the attached accelerometer.

3.2 Identified elastic parameters

From the experimental natural frequencies and mode shapes shown in Fig. 5 and Fig. 6, the elastic parameters for the lamina of the laminated circular cylindrical shells were estimated by the proposed inverse analysis method. The computations were carried out using the FEM eigenvalue program and the triangular shell element. Each shell is discretized

Table 3 Identified elastic parameters of lamina

Type of shell	Length l [mm]	E_L [GPa]	E_T [GPa]	G_{TV} [GPa]	G_{VL} [GPa]	G_{LT} [GPa]
Angle-ply laminated shell	253	111	7.63	2.00	5.19	5.19
	203	108	7.00	2.00	5.84	5.84
	151	103	7.15	2.00	5.39	5.39
Cross-ply laminated shell	253	119	7.40	1.90	5.50	5.50
	203	108	7.72	2.00	5.31	5.31
	153	110	7.38	2.01	5.11	5.11

Table 4 Elastic parameters of lamina obtained by experiment

E_L [GPa]	E_T [GPa]	G_{LT} [GPa]	ν_{LT}
95.4	6.35	5.22	0.32

with 1152 elements and 612 grid points. In the numerical calculations, the mass of the accelerometer was not considered because the mass of the accelerometer is very small. The identified elastic parameters of the lamina are shown in Table 3. In Table 3, the elastic moduli E_L, E_T in the direction of the parallel and normal to the fiber, shear moduli G_{TV}, G_{VL} and G_{LT} are shown (see Fig. 1). Poisson's ratios ν_{TV} and ν_{LT} are 0.32, respectively. From this table, one can find the good agreements between the identified elastic parameters for the lamina of the angle-ply laminated circular cylindrical shell and ones of the cross-ply laminated circular cylindrical shell. However, the identified elastic parameters for the lamina of each laminated circular cylindrical shell were slightly different. The possible explanation for this slight discrepancy is to apply the average of thickness for the each laminated circular cylindrical shell to the identifying calculation.

3.3 Justification of identified elastic parameters

To justify the identified elastic parameters for the lamina of these shells, elastic parameters of the specimens were obtained experimentally. The beam type specimens with fiber orientation angles of 0°, 45°, and 90° were cut from the prepreg sheet which was used as the lamina to make the laminated shell. The width and thickness of the specimens are 0.025 [m] and 0.42×10^{-3} [m], respectively. And the specimens of various lengths (0.10 [m] $\leq l \leq$ 0.28 [m]) were used. The elastic moduli E_L, E_T, and G_{LT} were estimated numerically from the fundamental natural frequencies obtained by free bending vibration tests of the cantilevered specimen. The Poisson ratio ν_{LT} was estimated from the tensile test of the specimen with the fiber orientation angle of 0°. The measured material properties of the lamina are listed in Table 4. From the comparison between Table 3 and Table 4, one can see the good agreements between experimental and identified elastic parameters. To confirm the identified elastic parameters, the FEM eigenvalue analysis was carried out by using the identified results. For the computation of natural frequencies and mode shapes, the mass of accelerometer is neglected because of it is very small compared to the shell's mass. Figures 7 and 8 show the natural frequencies and mode shapes of these shells estimated by the eigenvalue analysis. From Figs. 5 and 7, one can see that the difference between the experimental and the numerically calculated natural frequencies is about 1.3% at the most and one can find the excellent agreements between these mode shapes of the angle-ply laminated circular cylindrical shell. Also, Figs. 6 and 8 show the good agreements for the natural frequencies and mode shapes of the cross-ply laminated

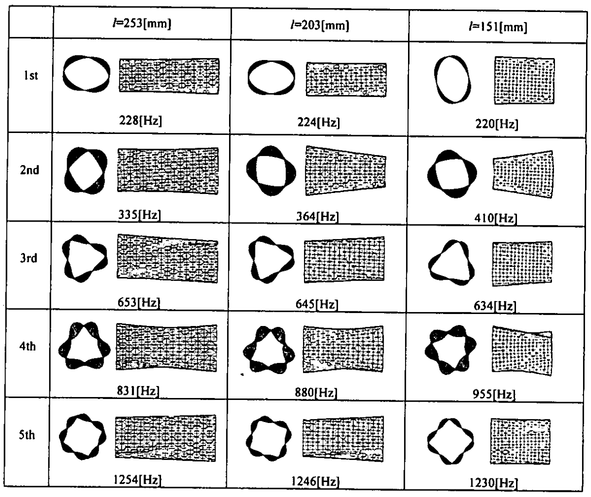

Fig. 7 Numerical natural frequencies and mode shapes of angle-ply laminated circular cylindrical shells; [30°$_2$/−30°$_2$/−30°$_2$/30°$_2$]

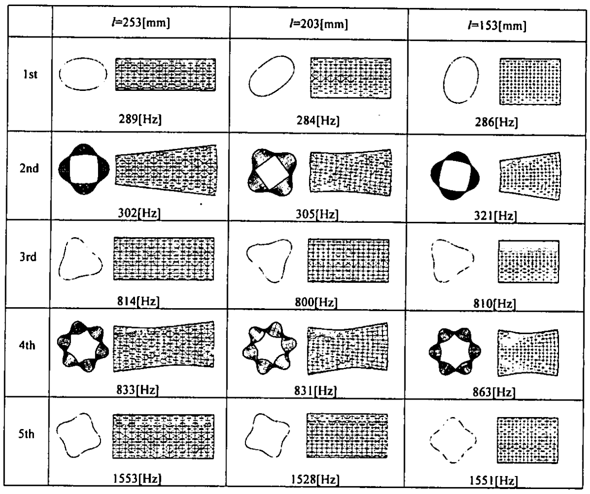

Fig. 8 Numerical natural frequencies and mode shapes of cross-ply laminated circular cylindrical shells; [0°$_2$/90°$_2$/90°$_2$/0°$_2$]

circular cylindrical shell.

4. Conclusions

The inverse analysis method to identify elastic parameters of the laminated composite materials was applied to the angle-ply and the cross-ply laminated circular cylindrical shells with free boundary conditions. On the other hand, the elastic parameters for the lamina of these shells were obtained experimentally. From the comparison between identified and experimental elastic parameters of the lamina, one can see the good agreements between these elastic parameters. Also, to confirm the identified elastic parameters, the FEM eigenvalue analysis was carried out by using the which ones. From the results, one can see the good agreements with respect to the natural frequency and mod shape. Accordingly, it follows that one can accurately estimate elastic parameters for the lamina of the laminated circular cylindrical shells by using the inverse analysis method proposed by the authors.

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