

Fractal Characteristics of Free Surface Profiles of Metal Sheets under Equi-Biaxial Tension*

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Free surface profiles of aluminum sheets under equi-biaxial tension are examined by employing three kinds of fractal analyses, i.e., the zeroset, power spectrum and box-counting methods. With an increase in plastic strain, long-wavelength components of the surface profiles become dominant, and their fractal structure tends to become constant beyond a certain strain. It is found that both surface roughness and fractal dimensions depend on the equivalent strain, independent of the stress ratio. A method for simulating the surface roughening behavior is presented by utilizing the power spectrum method.

Key Words: Plastic Forming, Surface Roughness, Press Working, Fractal, Self-Affinity, Sheet Metal, Free Surface, Equi-Biaxial Tension

1. Introduction

In sheet metal forming, the phenomenon of surface roughening due to plastic deformation is closely related to various surface problems, i.e., forming limit (necking caused by surface imperfection), coating feasibility, surface quality of products, galling and other friction characteristics. Thus, many studies on this phenomenon have been carried out, employing surface roughness as a characteristic measure and examining its relationship with plastic strain^{(1)~(3)}. However, the detailed geometry of a new surface formed by plastic deformation has not been investigated; thus, its actual state remains unclear. One of the reasons for this is that surface profiles involve various classes of randomness and are difficult to characterize quantitatively. In our previous research⁽⁴⁾ we established a fractal concept, and presented three analyti-

cal methods, thereby clarifying fractal characteristics on free surface profiles of aluminum sheets under uniaxial tension.

In the present study, using a similar approach to that used in the aforementioned research, fractals are shown on free surface profiles of aluminum sheets under equi-biaxial tension, and their dependence on the tensile method (stress ratio) and strain is examined. A method for simulating the variation of surface roughening with strain is also presented.

2. Determination of Fractal Dimensions

A fractal property is usually characterized by the fractal dimensions. In general, fractal dimensions of solid surfaces are regarded as a means of expressing an apparent complexity (randomness) in their geometries or of estimating their area. Although various methods for determining fractal dimensions had been attempted in past investigations, no general methods have been established, and theoretical relations among obtained dimensions remain to be solved. Accordingly, fractal dimensions are determined tentatively in the present study, based on three kinds of analyses presented in the previous investigation⁽⁴⁾. Here, it should be noted that in general, solid surfaces are not self-similar, but self-affine. Nonuniform scaling, where shapes are invariant under

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transformations that scale different coordinates by different amounts, is known as self-affinity. While the zero-set and power spectrum methods to be mentioned in the following are regarded as applicable to self-affine surfaces, the box-counting method, which has been employed in conventional applications, is based on a self-similarity assumption and is inadequate for use with solid surfaces⁽⁵⁾. However, this method is still convenient for estimating relative complexity of surfaces and is therefore also used here.

2.1 Zero-set method

Intersections of surface irregularities and the basal plane, referred to as zero-set elements here, are generated and their shapes are directly observed, as shown in Fig. 1. Even if surfaces are self-affine, if they are isotropic in-plane, the resultant zero-set elements become self-similar and reduce their fractal dimensions by one⁽⁶⁾. When the area and the peripheral length of each zero-set element are denoted by A_z and L_z , respectively, and L_z has the dimension D_z , the following is obtained from the measure-dimension relation⁽⁷⁾.

$$A_z^{1/2} \propto L_z^{1/D_z} \quad (1)$$

When the relation between L_z and A_z is measured experimentally, the zero-set dimension D_z can be determined as $D_z = 2/\alpha_z$ from the slope of the $\log(L_z)$ vs. $\log(A_z)$ plot, denoted by α_z .

2.2 Power spectrum method

The power spectra S_P and the wavelengths λ are obtained by applying FFT analysis to the surface profile curves, and the relation $S_P \propto \lambda^{-\beta}$ is assumed. The power spectrum dimension D_{PS} can be determined from the slope of the $\log(S_P)$ vs. $\log(\lambda)$ plot, denoted by β , as follows⁽⁸⁾:

$$\left. \begin{aligned} D_{PS} &= 2, & 0 \leq \beta < 1 \\ D_{PS} &= E + (3 - \beta)/2, & 1 \leq \beta \leq 3 \\ D_{PS} &= 1, & 3 < \beta \end{aligned} \right\} \quad (2)$$

where E is the Euclidean dimension, and $E=1$ in this study.

2.3 Box-counting method

The surface profile curves are closely covered with boxes of specified shapes and sizes⁽⁹⁾. While the shapes of boxes are similar, their sizes are varied (52 kinds of boxes were used in this study). The following relation is assumed between the covered box number N and its side length r .

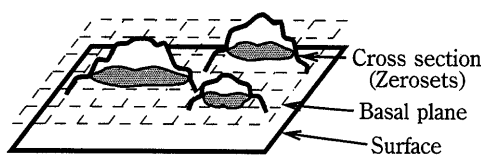


Fig. 1 Schematic explanation of zero-set method

$$N \propto r^{-D_B} \quad (3)$$

The box dimension D_B can be determined as $D_B = -\alpha_B$ from the slope of the $\log(N)$ vs. $\log(r)$ plot, denoted by α_B . Since the D_B value depends on the box shape, two shapes, square and rectangular, were employed; their width vs. height ratios were selected as 1 : 1 and 1 : 2.36 on the graphic plane, respectively.

3. Experimental Procedure

3.1 Material and equi-biaxial tension test

The tested materials were commercially pure aluminum sheets (A 1100-O, 0.8 mm thick, crystal grain size 17 μm). Their uniaxial properties are shown in Table 1. The initial surface roughnesses of the sheets (maximum height roughness R_y) were 0.53 μm and 2.13 μm in the directions parallel and perpendicular to the rolling direction, respectively. The outline of the equi-biaxial tension test is illustrated in Fig. 2. While circular plates 100 mm in diameter were machined from the original aluminum sheets, circular driving plates of the same size were made of killed steel sheets (0.8 mm thick) and in each of these sheets a 10-mm-diameter hole was bored at the center. After the driving plate was put on the aluminum plate (specimen), both were stretch-formed with a flat-headed punch 36 mm in diameter and 4 mm in profile radius. The interface between the punch and the driving plate was lubricated with graphite grease. The stretch-forming test was carried out by means of the sheet metal forming machine under 0.2 mm/s punch speed. These conditions were found to be

Table 1 Uniaxial properties of A 1100

Direction	Tensile strength MPa	N-value*	F-value* MPa	r-value	Total elongation %
0°	103	0.27	193	0.66	27.1
45°	95	0.27	176	1.18	32.8
90°	99	0.28	187	0.66	33.1
Mean	99	0.27	185	0.92	31.5

$$*: \sigma = F \varepsilon^N$$

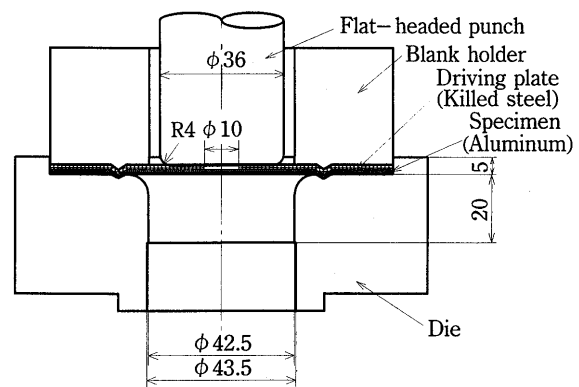


Fig. 2 Equi-biaxial tension test

sufficient for deforming the flat head region of specimens until the initiation of localized necking.

3.2 Generation and measurement of zeroset elements

In order to estimate the zeroset dimension D_z , small square pieces of 12 mm side length were cut out from the flat head region of specimens, and their lower free surfaces (see Fig. 2) were lapped on a precision lapping plate ($0.04 \mu\text{m } R_y$) with fine diamond powder ($0.125 \mu\text{m}$ nominal grain size); thus, the zeroset elements were generated. Extreme care was taken not to cut the valley walls of surface irregularities during lapping. The zeroset elements were observed at the central region of the test pieces within a circle of 10 mm diameter through an optical microscope and a CCD camera. After signals of the magnified images were transferred to an image processor and processed into binary images, the area A_z and the peripheral length L_z of each zeroset element were measured. One picture involved 512×512 pixels, and each pixel was calibrated as $2.33 \times 2.33 \mu\text{m}^2$.

3.3 Measurement of surface profile curves

In order to estimate the dimensions D_{PS} and D_B , surface profile data were obtained under the no cutoff condition by means of a stylus profilometer (Talysurf 10), and their signals were digitized and transferred to a computer. The stylus end surface was flat and square ($2.5 \times 2.5 \mu\text{m}^2$). The resolution of this measuring apparatus was estimated under the nominal magnifications of $\times 1000$ and $\times 20$ as $0.0174 \mu\text{m}$ and $2.054 \mu\text{m}$ in the vertical and traverse directions, respectively. Measurement of the profile curves was carried out three times for each of the two directions, parallel and perpendicular to the rolling direction, on the lower free surface of the specimen shown in Fig. 2. These respective directions will be denoted by 0° and 90° hereafter.

4. Results and Discussion

4.1 Surface roughening behavior

Surface curves recorded in the 90° direction are exemplified in Fig. 3. In the present experiment, when the equivalent strain ϵ_{eq} exceeded 0.71, a trough with a width near the sheet thickness appeared on the specimen surface, parallel to the 0° direction. Accordingly, this value was regarded as the critical strain of necking initiation. From Hill's anisotropic theory, the equivalent strain ϵ_{eq} in equi-biaxial tension is expressed as $\epsilon_{eq} = \sqrt{(1+r)/2} |\epsilon_t|$, where r is the anisotropic parameter called an r -value, and ϵ_t is the thickness strain. Referring to Table 1 and using the in-plane mean r -value of 0.92, the relation $\epsilon_{eq} = 0.97 |\epsilon_t|$ is obtained; thus, the difference between ϵ_{eq} and $|\epsilon_t|$ is small. Furthermore, it was pointed out that Hill's

anisotropic theory is inadequate for aluminum sheets⁽¹⁰⁾. For these reasons, the equivalent strain was estimated by $\epsilon_{eq} = |\epsilon_t|$, assuming an isotropic material.

Variations of surface roughness (maximum height R_y) with tensile strains ϵ_u (uniaxial) and $\epsilon_b (= |\epsilon_t|/2$, equi-biaxial) are shown in Fig. 4(a), where R_y values in uniaxial tension were measured for the specimens elongated in the 0° direction. It is seen from Fig. 4(a) that within a certain strain level the R_y value increases linearly with increasing strains under the respective tensile methods (stress ratios). However, the construction of R_y vs. ϵ_{eq} plots gives a single relation including two tensile methods, except for the steep increase in R_y due to the occurrence of necking, as shown in Fig. 4(b). Such a result that the dependence of R_y on the strain can be expressed as a

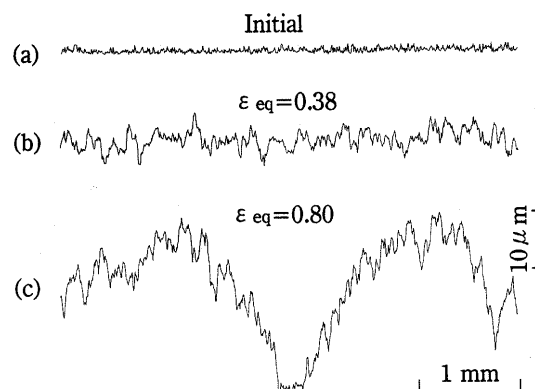
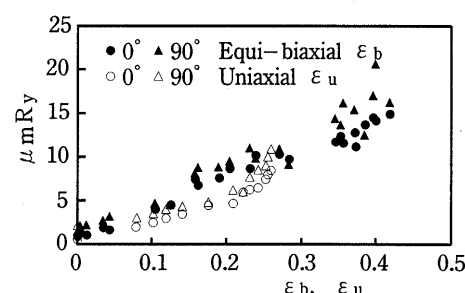
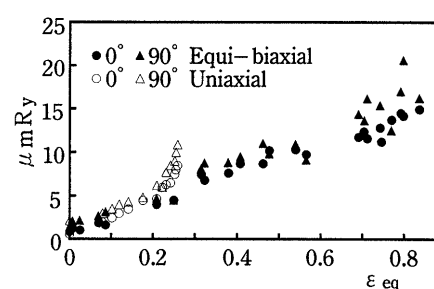


Fig. 3 Examples of recorded surface profiles (90° direction)



(a) R_y vs. ϵ_b and ϵ_u



(b) R_y vs. ϵ_{eq}

Fig. 4 Relationship between surface roughness R_y and various strains

unified relation irrespective of the stress ratio by employing ε_{eq} was reported previously by Yamaguchi et al⁽²⁾.

4.2 Zeroset dimension

Images of the zeroset elements obtained in the equi-biaxial tension test are exemplified in Fig. 5, where the white parts correspond to the zeroset elements. The relative cutting height denoted by δ is defined as $\delta = (R_{y0} - R_{yc}) / R_{y0} \times 100(\%)$, where R_{y0} and R_{yc} are the surface roughness before and after cutting, respectively. An example of the log-log plots of the peripheral lengths L_z vs. areas A_z measured for the zeroset elements is shown in Fig. 6, where a linear relation is observed, indicating that the zeroset elements follow a fractal geometry. In this study, considering pixel resolution, the zeroset dimension D_z

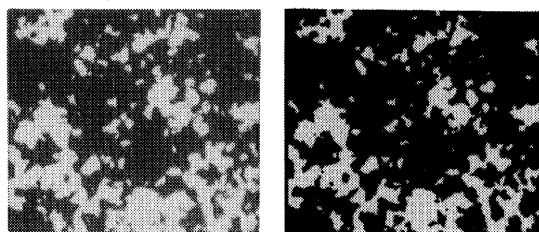


Fig. 5 Example of image of zeroset plane ($\varepsilon_{eq}=0.44$, $\delta=36.8\%$)

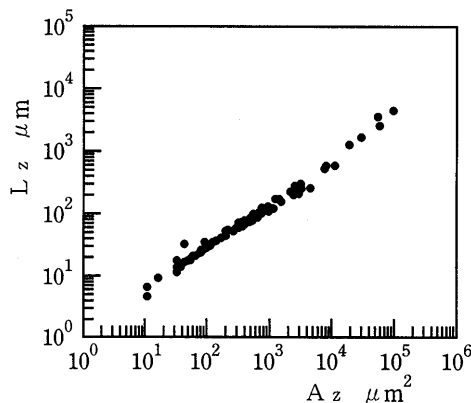


Fig. 6 Relationship between peripheral length L_z and area A_z of zeroset elements ($\varepsilon_{eq}=0.44$, $\delta=36.8\%$)

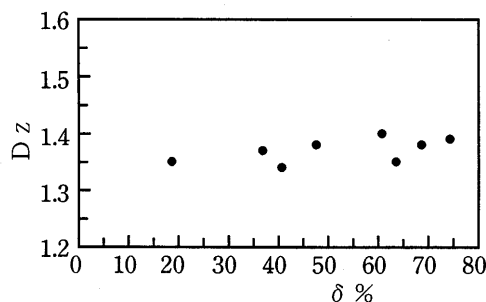


Fig. 7 Relationship between D_z and relative cutting height δ ($\varepsilon_{eq}=0.44$)

was determined in the range $A_z \geq 20 \mu\text{m}^2$. The relation between D_z and δ is shown in Fig. 7, indicating that D_z is nearly constant and independent of δ ; thus, each asperity of the surface has a uniform fractal structure. Mean values of D_z for 6 - 8 kinds of δ are denoted by \bar{D}_z , and their relation with equivalent strain ε_{eq} is shown in Fig. 8. \bar{D}_z decreases with increasing ε_{eq} and attains a nearly constant value of 1.35 beyond $\varepsilon_{eq} \approx 0.1$. \bar{D}_z values in the case of uniaxial tension are added to Fig. 8, and it is noted that \bar{D}_z can be plotted on a certain unique curve throughout the uniaxial and equi-biaxial tensions by using ε_{eq} . A similar result has been obtained with regard to R_y , as seen in the previous section.

4.3 Power spectrum dimension

Log-log plots of the power spectra S_P and wavelengths λ are exemplified in Fig. 9, where S_P values are expressed by the measured spectra divided by the measurement time (53.2 s). A linear relation which shows an inflection at $\lambda \approx 100 \mu\text{m}$ is observed; thus, S_P follows a multifractal property. Similar phenomena to this were always seen irrespective of the measurement direction of surface profile curves and the strains. The transitional wavelength at the inflection point was found to be nearly constant ($\lambda \approx 100 \mu\text{m}$), but the reason for this is unclear at present. In the range of λ longer than $100 \mu\text{m}$, denoted by the region

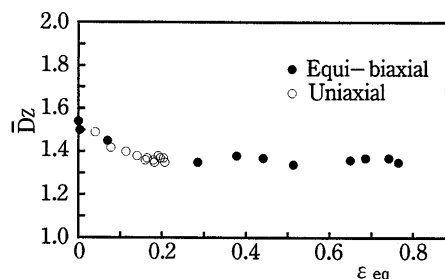


Fig. 8 Relationship between \bar{D}_z and equivalent strain ε_{eq}

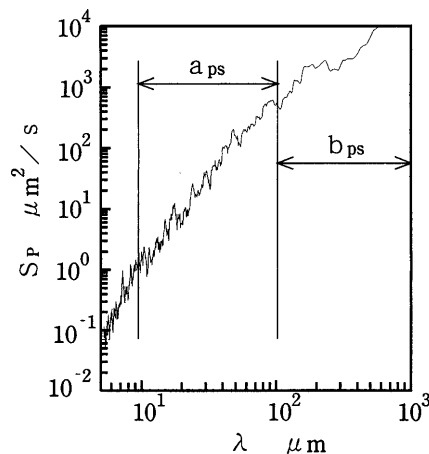


Fig. 9 Relationship between power spectrum S_P and wavelength λ (90° , $\varepsilon_{eq}=0.57$)

b_{PS} in Fig. 9, $\beta \leq 1$ ($D_{PS}=2$) was always found, indicating that a variation in β has no effect on D_{PS} . A limit of the shorter wavelength was specified as $10\text{ }\mu\text{m}$, taking into account an error due to the geometry of the stylus profilometer. Finally, the power spectrum dimension D_{PS} was estimated for λ ranging from $10\text{ }\mu\text{m}$ to $100\text{ }\mu\text{m}$.

The relation between D_{PS} and ε_{eq} is shown in Fig. 10. D_{PS} decreases with increasing ε_{eq} , and attains a nearly constant value of 1.1 beyond $\varepsilon_{eq}=0.1$ in the 0° direction or beyond $\varepsilon_{eq}=0.3$ in the 90° direction. With an increase in ε_{eq} , the difference between D_{PS} values in the two directions decreases, suggesting that the fractal of surface profiles tends to be in-plane isotropic.

4.4 Box dimension

Log-log plots of the covered box number N and the box width r , obtained from the surface profile curves, are shown in Fig. 11, where a linear relation showing an inflection at the I.P. point is observed again, presenting an apparently multifractal character. Such multifractal phenomena were always observed irrespective of the box shape, measurement direction and strain. The slope of the line in region b_B was found to

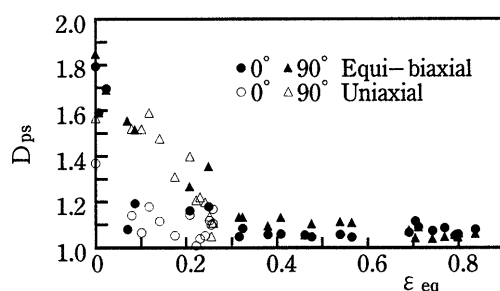


Fig. 10 Relationship between D_{PS} and equivalent strain ε_{eq}

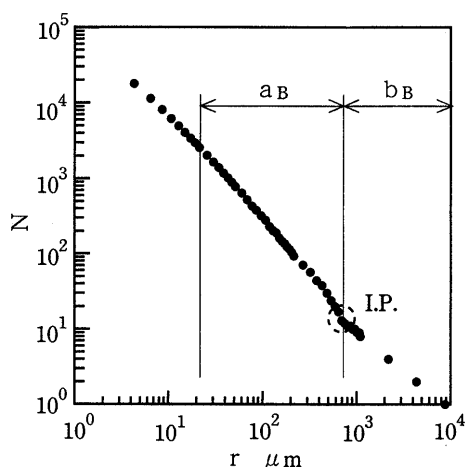


Fig. 11 Relationship between box number N and box width r in box-counting method (90° , $\varepsilon_{eq}=0.21$, rectangular box)

be always near -1 ($D_B \approx 1$). This is peculiar to the box-counting method, where in the range of r exceeding a certain limit, details of the surface profiles cannot be detected and are estimated as smooth. Taking into account an error due to the geometry of the stylus profilometer, box dimension D_B was determined in region a_B with r ranging from $10\text{ }\mu\text{m}$ to the I.P. point.

The relation between D_B and equivalent strain ε_{eq} is shown for the case of rectangular boxes in Fig. 12. While D_B increases in the 0° direction with increasing ε_{eq} , it decreases in the 90° direction, and then both D_B values become nearly constant beyond $\varepsilon_{eq} \approx 0.3$. The difference in D_B between the two directions reduces with increasing ε_{eq} , and here also, the fractal tends to be in-plane isotropic. Although not shown here, the D_B value in the case of the square box was found to be less than that in the case of the rectangular box, but its variation with ε_{eq} was similar to the aforementioned.

The box width at the aforementioned I. P. point is denoted by r_{cr} , and its relation with ε_{eq} is shown in Fig. 13. It is noted that r_{cr} increases linearly with increasing ε_{eq} , followed by a steep increase. Strain ε_{eq} upon the steep increase in r_{cr} is about 0.7 in equi-biaxial tension, and nearly agrees with that on localized necking initiation. A similar phenomenon to this

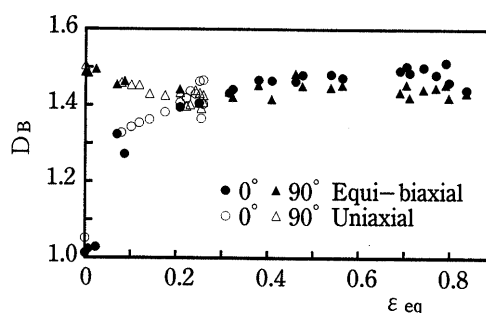


Fig. 12 Variation of D_B with equivalent strain ε_{eq} (rectangular box)

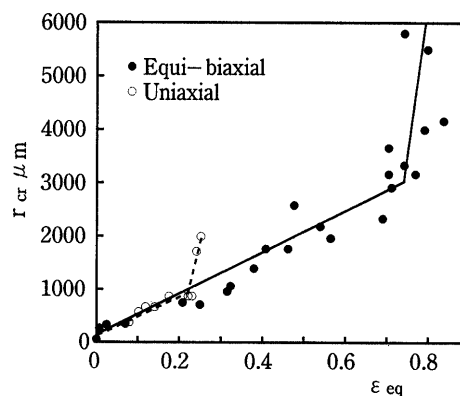


Fig. 13 Variation of critical box width r_{cr} with equivalent strain ε_{eq} (90° , rectangular box)

was also observed in the uniaxial tension test⁽⁴⁾. Such a steep increase in r_{cr} is attributable to large surface waviness caused by localized necking (see Fig. 3). Although R_y was also expected to indicate necking initiation, it did not give as good a reproducibility and as clear a result as r_{cr} did.

4.5 Characteristics of surface roughening and their dependence on stress ratios

Various fractal dimensions obtained in the present experiment are summarized in Fig. 14. Results in the uniaxial tension are also added. \bar{D}_{PS} and \bar{D}_B are averages of D_{PS} and D_B in the 0° and 90° directions, respectively. They as well as \bar{D}_Z are regarded as means for expressing in-plane mean properties, and are focused on in the following discussion.

As discussed previously⁽⁴⁾, while \bar{D}_Z is a measure that expresses directly the complexity in surface geometries, \bar{D}_{PS} indicates the dependence of power spectra on the wavelength; thus, it is taken to be an indirect measure, because it involves the transformation of surface geometries into the power spectra. \bar{D}_B , which is easy to estimate, is also a measure related to surface complexity, and its property may be rather near to that of \bar{D}_Z . However, since \bar{D}_B attributes no unique value to self-affine surfaces⁽⁵⁾, it should be used for an expedient purpose. The different fractal dimensions shown in Fig. 14 have different physical meanings. However, characteristics common to all the fractal dimensions are as follows:

- ① Employing the equivalent strain ε_{eq} facilitates expression of strain dependence of fractal dimensions as a single curve, irrespective of the tensile method (stress ratios).
- ② Either of the fractal dimensions becomes constant beyond a certain strain level.

Characteristic ① is also seen in Figs. 10 and 12. Yamaguchi et al.⁽²⁾ carried out tensile tests while varying strain ratios, and obtained a similar conclusion with regard to surface roughness R_y . These results suggest that if macroscopic plastic work on the sheets is specified, their corresponding surface

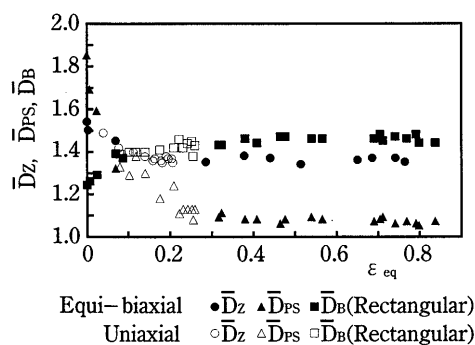


Fig. 14 Comparison among various fractal dimensions

structure is uniquely determined independent of the stress ratio. However, further investigations are required to confirm whether this rule holds in general. Characteristic ② indicates that when a surface area increases macroscopically and microscopically with increasing plastic strain and is eventually dominated by the newly formed surface, the fractal structure of surfaces becomes constant.

The \bar{D}_Z value in the steady state is found to be about 1.35. Since it is a dimension of the periphery curves of zero-set elements generated from a surface, the dimension of this original surface can be estimated as 2.35, because the surface dimension is larger by one than that of the periphery curves, as stated in section 2.1. The decrease in \bar{D}_{PS} seen before the steady state means that with an increase in surface area, spectrum components with longer wavelengths become dominant. The information on \bar{D}_{PS} is utilized for simulation of surface roughening, as mentioned in the following.

5. Simulation of Surface Roughening

An attempt was made to develop a computer simulator creating surface profiles by modeling a distribution of the power spectrum and by using inverse FFT. A model of the spectrum distribution is schematically illustrated in Fig. 15. First, the relation of $\log(S_p)$ vs. $\log(\lambda)$ was approximated to three broken lines, referred to as a base spectrum S'_p here. The parameter λ_1 is a wavelength at the boundary between regions a_{PS} and b_{PS} shown in Fig. 7, and a straight line was applied to the range $\lambda \leq \lambda_1$. From an examination in the range $\lambda > \lambda_1$ (region b_{PS}), an approximation to two broken lines was regarded as adequate for the purpose. The slopes of three lines and the wavelengths at the inflection points are denoted by $\beta_1, \beta_2, \beta_3$ and λ_1, λ_2 in Fig. 15, respectively.

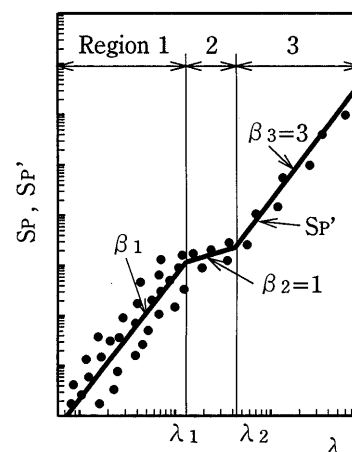


Fig. 15 Three-broken-lines model of power spectra under equi-biaxial tension

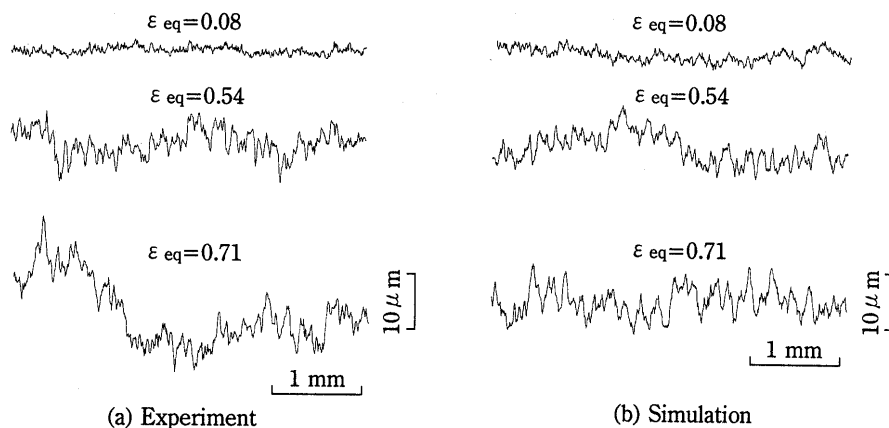


Fig. 16 Comparison of surface profiles between experiment and simulation (90°)

Second, using the following equation, the base spectrum S_P^* was transformed into the spectrum S_P involving such a random dispersion as shown in Fig. 9.

$$S_P = S_P^* G^2 = 10^c \lambda^{\beta} G^2 \quad (4)$$

where G is Gaussian random number, and c and β are constants in each line constituting the base spectrum. The real and imaginary parts of S_P , denoted by A and B , were calculated using the following equations⁽¹¹⁾.

$$\begin{aligned} A &= S_P^{1/2} \cos(2\pi\phi) \\ B &= S_P^{1/2} \sin(2\pi\phi) \end{aligned} \quad (5)$$

where ϕ is a random number ranging from 0 to 1. After A and B were determined, surface profiles were generated through the inverse Fourier transformation. According to an examination of the input parameters, the simulation accuracy improved with their values near the experimental ones. However, good results were still obtained by fixing β_2 and β_3 as 1 and 3, respectively. Thus, the input parameters were reduced to four (c and β_1 in region 1, λ_1 and λ_2), and determined from the experimental data. Surface profiles created by the aforementioned method are compared with those obtained experimentally in Fig. 16, indicative of a satisfactory simulation except for the waviness accompanied by the initiation of localized necking. In this way, the validity of the present simulation has been confirmed. However, it may be said that this consequence is a matter of course, because the parameters used in the simulation have been evaluated directly by the experiment. However, although not shown here, by introducing some functions expressing the relations of the respective parameters c , β_1 , λ_1 and λ_2 with the equivalent strain ε_{eq} , prediction of a surface profile at any specified strain was achieved.

6. Concluding Remarks

The results obtained in the present research are summarized as follows:

- (1) In the process of surface roughening under

uniaxial and equi-biaxial tensions of aluminum sheets, surface profiles with long wavelength components grow with increasing surface area, followed by a steady state causing a constant fractal dimension. Accordingly, the fractal structure of newly formed surfaces due to plastic deformation is regarded as constant.

- (2) The increase in plastic strain reduces the variation of fractal dimensions in the tested directions, thus making surfaces in-plane isotropic.

- (3) The box-counting method serves to sensitively detect the initiation of localized necking in both uniaxial and equi-biaxial tensions.

- (4) As a possible rule with regard to the roughness and fractal dimension of roughening surfaces, it is suggested that their strain dependence is unifiable by employing the equivalent strain, irrespective of the tensile state (stress ratios).

- (5) Based on the results of power spectrum analysis, a method for simulating surface roughening is presented, and its validity is confirmed.

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